Motion Coordination of Groups of Car-like Robots ⋆

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Abstract: Formation, marching and inter-robot collision avoidance are fundamental problems in the motion coordination of mobile robots. The control strategies are decentralized because every robot does not possess all information about the positions and goals of the other robots and it enables collision avoidance laws only in the proximity of another robots. This paper extends our previous works dedicated to the case of motion coordination of omnidirectional point robots, to the case of the kinematic model of car-like mobile robots. It is demonstrated the convergence of the formation and marching strategies applies to any communication graph, using the properties of the Laplacian matrix and artificial vector fields. The approaches are tested by numerical simulations with a virtual reality environment.

Keywords: Mobile robots; Decentralized control; Formation control; Graph theory; Car-like robots

1. INTRODUCTION

Multi-agent Robots System (MARS) is a set of mobile agents that communicate between them to achieve a common task. In the last 30 years, the MARS coordination has found different applications in the fields of security, exploration, ethology, Automated Guided Vehicle in manufacturing systems, etc. [Cao et al., 1997]. The motion coordination extends the classic problems of point convergence, path following and collision avoidance for the case of one kinematic or dynamical model to the case of multiple agents [Arai et al., 2002]. The group convergence now is analyzed as formation control, and the group path following is addressed as formation tracking, flocking behavior or marching control. The inter-robot collision avoidance now is added to the analysis considering reactive strategies in the proximity of each robot [Chen and Wang, 2005].

The control strategies can be classified in two categories. The behavior-based strategies, copies the behavior of biological entities for the design of simple reactive rules where the robots are aggregated to formation patterns according to the relative distances between neighbors [Balch and Arkin, 1998]. This is the case of the swarms literature, where the robots does not have a specific position in the group and the main feature is the analysis of the consensus, formation merging and splitting, and other phenomena appeared in the social behavior of multiple robots. On the other hand, in the formation graph schemes, each robot communicates with other robots according to a specific topology of communication. Thus, the global convergence is analyzed using the properties of the Laplacian matrix of the communication graph [Desai, 2002]. In this approach, all the robots have a specific position in the formation, with possible group roles like leader or follower. The global information is not available for all the group, becoming a decentralized configuration [Muhammad and Egerstedt, 2004]. An study of the art of formation graph and the technique of artificial vector fields for formation control and collision avoidance was presented in [Hernández-Martínez and Aranda-Bricaire, 2011].

Because the main objective of formation and marching control is the group convergence and stability, the majority of the authors studies the case of point robots or omnidirectional robots, without considering the nonholonomic motion restrictions and simplify the analysis. Some related works are [Dimarogonas and Kyriakopoulos, 2006] and [Hernández-Martínez and Aranda-Bricaire, 2012]. For the case of wheeled motion robots, the formation control was developed firstly for the case of unicycle-type robots [Desai et al., 2001], including exact linearizing, backstepping, nonlinear control, etc. The other case of nonholonomic robot is the car-like configuration, where the study of coordination has been bounded to the case of leader-follower configuration or open chain configuration. Some related works are [Pereira et al., 2003], where a reactive control laws are designed in a limited workspace with an omnidirectional camera used as position sensor. A formation between a pair leader-follower based on the distance and relative angle is studied in [Gil-Pinto et al., 2007], where an algorithm is constructed for the path following. The work in [Xu et al., 2012] adds a PD-type controller to compensate the delay effect in the communication. Lyapunov control for two car-like robots is proposed in [Ramaswamy and Balakrishnan, 2008], that combines a velocity controller, a torque controller and a neural network to achieve the formation of a follower respect to a leader, computing online the friction and desired accelerations. The collision avoidance

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in a minimum time optimal control problem is addressed in [Saska et al., 2009], where the robots achieve a target in a group formation. Finally, [Ronen and Arogeti, 2012] simplifies the car-like robot to the bicycle kinematic model, where the geometric formation defines some trajectories for each robot, and the time derivative of the path is utilized to ensure the convergence of the formation errors. All the previous works does not analyze the general case of formation with an arbitrary formation graph and the bounded of the steering angle of the car-like robot, that generates singularities in the control law [Brockett, 1983].

In this paper, we extend the formation and marching control laws based on formation graphs previously studied in [Hernández-Martínez and Aranda-Bricaire, 2012] and [Hernandez-Martinez et al., 2013], respectively, for the case of point robots, to the case of the kinematic model of a car-like robot. An appropriated control output is selected to linearize the system and to achieve any well-defined graph of communication. The paper also shows an example of the restriction of the steering of the robots and their influence in the group convergence. The approach is tested in a virtual reality platform to show the performance of the robot's trajectories.

The paper is organized as follows. In the Section 2 is presented the kinematic model and the formation graph definitions. In the Section 3 the formation and marching control laws, respectively, are established for the case of the car-like robots. Finally, the Section 5 presents some conclusion remarks and future work.

2. PROBLEM DEFINITION

2.1 Kinematic model of the car-like robot

Based on [Canudas de Wit et al., 2012], the kinematic model of the car-like robot (see Figure 1) is presented. The development of the model is based on a three wheel (W_1, W_2, W_3) mobile robot of type $(1,1)$. The front steering wheel (W_3) angle is used to calculated the actual steering angles for the cart-like front wheels using the Ackermann's steering relations. [Siegwart and Nourbakhsh, 2004, Hrbáček et al., 2010]. Consider the rotation

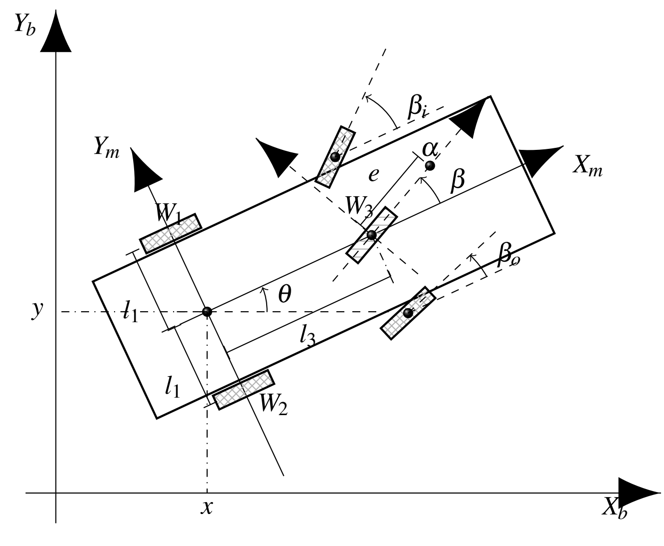

Fig. 1. Mobile robot car-like

matrix between the X_b-Y_b frame and the X_m-Y_m frame be given by

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Let $\xi = (x \ y \ \theta)^\top$ be the vector of the position and orientation of the middle point of the rear axis in Figure 1. Then the kinematic posture model for a car-like mobile robot is given by:

$$\dot{\xi} = R^\top(\theta)\Sigma(\beta)v = \begin{pmatrix} l_3\cos\theta\cos\beta \\ l_3\sin\theta\cos\beta \\ \sin\beta \end{pmatrix} v \tag{2}$$

$$\dot{\phi} = J_2^{-1} J_1 \Sigma(\beta) v \tag{3}$$

$$\cot\beta_o = \cot\beta + \frac{2l_1}{l_3} \tag{4}$$

$$\cot\beta_i = \cot\beta - \frac{2l_1}{l_3} \tag{5}$$

with

$$\Sigma(\beta) = \begin{pmatrix} l_3\cos\beta \\ 0 \\ \sin\beta \end{pmatrix}$$

and

$$J_1 = \begin{pmatrix} 1 & 0 & -l_1 \\ 1 & 0 & -l_1 \\ \cos\beta & \sin\beta & l_3\sin\beta \end{pmatrix}, \quad J_2 = rI,$$

$$C_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ -\sin\beta & \cos\beta & l_3\cos\beta \end{pmatrix}, \quad \dot{\phi} = \begin{pmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{pmatrix}.$$

where r is the radius of the wheels, $\dot{\phi}_i$ is the angular velocity for the different wheels. The relation between the angles β, β_o and β_i is given by the Ackermann steering assumption (4–5). This type of mobile robot presents a singular point when $\beta = \pm\pi/2$. Moreover, usually the arc that the steering is allowed to have is also restricted such that $|\beta| \leq b_*$, where b_* depends on the maximal allowed excursion for β_i and β_o

The kinematic posture model described around the center of the rear axis presents a singularity that can be avoided if instead the point α is used for control. The coordinates of α are

$$\alpha = \begin{pmatrix} x + l_3\cos\theta + e\cos(\theta+\beta) \\ y + l_3\sin\theta + e\sin(\theta+\beta) \end{pmatrix} \tag{6}$$

and its time derivative

$$\dot{\alpha} = \begin{pmatrix} \dot{x} - l_3\dot{\theta}\sin\theta - e\left(\dot{\theta}+\dot{\beta}\right)\sin(\theta+\beta) \\ \dot{y} + l_3\dot{\theta}\cos\theta + e\left(\dot{\theta}+\dot{\beta}\right)\cos(\theta+\beta) \end{pmatrix} \tag{7}$$

that is

$$\dot{\alpha} = A(\theta,\beta,l_3,e)\begin{pmatrix} v \\ u \end{pmatrix} \qquad \dot{\beta} = u \tag{8}$$

with

$$A(\theta,\beta,l_3,e) = \begin{pmatrix} l_3\cos(\theta+\beta) - e\sin\beta\sin(\theta+\beta) & -e\sin(\theta+\beta) \\ l_3\sin(\theta+\beta) + e\sin\beta\cos(\theta+\beta) & e\cos(\theta+\beta) \end{pmatrix} \tag{9}$$

Observe that since

$$\det A(\theta,\beta,l_3,e) = el_3 \tag{10}$$

the matrix $A(\theta,\beta,l_3,e)$ is invertible as long as $l_3 \neq 0$ and $e \neq 0$.

In the following, the equations for α, (8), will be used to achieve formation and marching control.

3. FORMATION AND MARCHING CONTROL

According to [Desai, 2002, Hernández-Martínez and Aranda-Bricaire, 2012], the desired communication of a group of robots can be represented by a Formation Graph (FG) defined as follows. Consider $N = \{\alpha_1,...,\alpha_n\}$ the set of front points of a team of $R_1,...,R_n$ car-like mobile robots, respectively. Let $N_i \subset N$ the subset of robots, detectable from robot R_i, then

Definition 1. The Formation Graph $G = \{Q,E,C\}$ is a graph that consists on (i) a set of vertices $Q = \{R_1,R_2,...,R_n\}$ related to the team members, (ii) a set of edges $E = \{(j,i) \in Q \times Q\}$, $i \neq j$ containing pairs of nodes that represent inter-agent communications, therefore $(j,i) \in E$ iff $j \in N_i$ and (iii) a set of vectors $C = \{c_{ji}\}$, $\forall (j,i) \in E$ that specify the desired relative position between agents i and j, i.e. $\alpha_i - \alpha_j = c_{ji} \in \mathbb{R}^2$, $\forall i \neq j, j \in N_i$ within a desired formation pattern.

A well-defined FG must satisfy the next conditions: (1) the graph is connected, i.e. there are no isolated nodes, (2) there are no conflict in the desired position vectors, in the sense that if $c_{ij}, c_{ji} \in C$, then $c_{ij} = -c_{ji}$ and consequently, (3) the desired position vectors establish a closed-formation, i.e., if there exist vectors c_{jm_1}, $c_{m_1m_2}$, $c_{m_2m_3}$,..., c_{m_rj}, then they must satisfy:

$$c_{jm_1} + c_{m_1m_2} + c_{m_2m_3} + ... + c_{m_rj} = 0. \quad (11)$$

The previous condition establishes that the vectors form closed-polygons and it is related to the existence of equilibria.

For a given set of n mobile car-like robots equations with respect to α_i

$$\begin{aligned} \dot{\alpha}_i &= A_i(\theta_i,\beta_i,l_{3_i},e_i)\begin{pmatrix} v_i \\ u_i \end{pmatrix}, \quad i = 1,...,N \\ \dot{\beta}_i &= u_i \end{aligned} \quad (12)$$

The formation control is achieved by the control law

$$\begin{pmatrix} v_i \\ u_i \end{pmatrix} = -\frac{k}{2}A_i^{-1}(\theta_i,\beta_i,l_{3_i},e_i)\frac{\partial \gamma_i}{\partial \alpha_i} \quad (13)$$

with γ_i the local potential function

$$\gamma_i = \sum_{j\in N_i} \left|\alpha_i - \alpha_j - c_{ji}\right|^2 \quad (14)$$

The marching control is achieved by the control law

$$\begin{pmatrix} v_i \\ u_i \end{pmatrix} = -\frac{k}{2}A_i^{-1}(\theta_i,\beta_i,l_{3_i},e_i)\left(\frac{\partial \gamma_i}{\partial \alpha_i} + \dot{m}(t)\right) \quad (15)$$

with $m(t) : \mathbb{R} \to \mathbb{R}^2$ is the parametrization of the marching trajectory with continuous first derivative.

$$\begin{aligned} \gamma_i &= \sum_{j\in N_i} \left|\alpha_i - \alpha_j - c_{ji}\right|^2, \; i = 1,...,n-1 \\ \gamma_n &= \sum_{j\in N_n} \left|\alpha_n - \alpha_j - c_{jn}\right|^2 + \left|\alpha_n - m(t)\right|^2 \end{aligned} \quad (16)$$

The closed-loop system for the case of the formation control (12)-(13) for the coordinates α_i can be reduced to

$$\dot{\alpha}_i = -k\sum_{j\in N_i}(\alpha_i - \alpha_j - c_{ji}), \; i = 1,...,n-1 \quad (17)$$

and for the closed-loop system for the marching control (12)-(15), now is reduced to

$$\begin{aligned} \dot{\alpha}_i &= -k\left(\sum_{j\in N_i}(\alpha_i - \alpha_j - c_{ji}) + \dot{m}(t)\right), \; i = 1,...,n-1 \\ \dot{\alpha}_n &= -k\left(\sum_{j\in N_n}(\alpha_n - \alpha_j - c_{jn}) + \alpha_n - m(t) + \dot{m}(t)\right) \end{aligned} \quad (18)$$

When the robots are working in the linear region, due to the decoupling matrix (9) the closed loop equations (17) and (18) for the vehicles are similar to the case of the point-robot for which convergence of vehicle trajectories to the desired trajectory has been shown in [Hernández-Martínez and Aranda-Bricaire, 2012] for the case of formation, and [Hernandez-Martinez et al., 2013] for the case of marching, for any well-defined formation graph.

Note that the closed-loop for the marching control law (18) reduces to the quasi formation control problem if $m(t)$ is a constant function. Furthermore, the error systems is a linear autonomous equation with only eigenvalues on the rigth half complex plane which ensures the exponential convergence.

Remark 1. The main contribution of this work is the extension of the control laws (13) and (15) for the case of car-like robots using the appropriated linearizing output (6). This combination of control laws and mobile robot type has not been reported in the literature as far as we know. The inter-robot collision avoidance strategies are not considered in this paper.

3.1 Control strategies with saturated values in the steering angle

Since $|\beta_i| < b_*$, an anti-windup control is included

$$\dot{\beta}_i = u_i - k_s * (\beta_i - \mathrm{sat}(\beta_i)) \quad (19)$$

$$\mathrm{sat}(\beta_i) = \begin{cases} -b_*, & \beta_i < -b_* \\ \beta_i, & -b_* \leq \beta_i \leq b_* \\ b_*, & \beta_i \geq b_* \end{cases} \quad (20)$$

It is expected that the basin of attraction for the equilibria to be reduced due to the saturation, so for a sharp enough or fast enough trajectory we expect that the marching control will not converge to the desire trajectory. The study of the design of trajectories that derives in an all-time convergence for a given value of saturation can be addressed in further research.

4. SIMULATION RESULTS

Here we present three simulations for the marching control for a three vehicle platoon.

For the first and second simulations the initial condition for the vehicles is as follows, $(x_1,y_1,\theta_1,\beta_1) = (-30,0,\frac{\pi}{10},0)$, $(x_2,y_2,\theta_2,\beta_2) = (0,30,-\frac{\pi}{10},0)$, $(x_1,y_3,\theta_3,\beta_3) = (30,0,\frac{3\pi}{10},0)$. The formation is triangular with $c_{21} = (0,11)$, $c_{32} = (16.5,-11)$, $c_{31} = (16.5,0)$ and the communications is complete, i.e., each vehicle communicates with the all the vehicles. The saturation restriction for the turning angle is $b_* = \frac{\pi}{8}$. The elliptical trajectory is parameterized as

$$m(t) = \left(30\cos\left(\frac{\pi}{60}t\right), 15\sin\left(\frac{\pi}{60}t\right)\right) \quad (21)$$

The controller and anti-windup gains are $k = 5$, $k_s = 8$ with $l_1 = 1.5$, $l_3 = 3$ and $e = 0.5$.

The first simulation is the marching control with trajectory given by (21) with a triangular formation and no saturation on

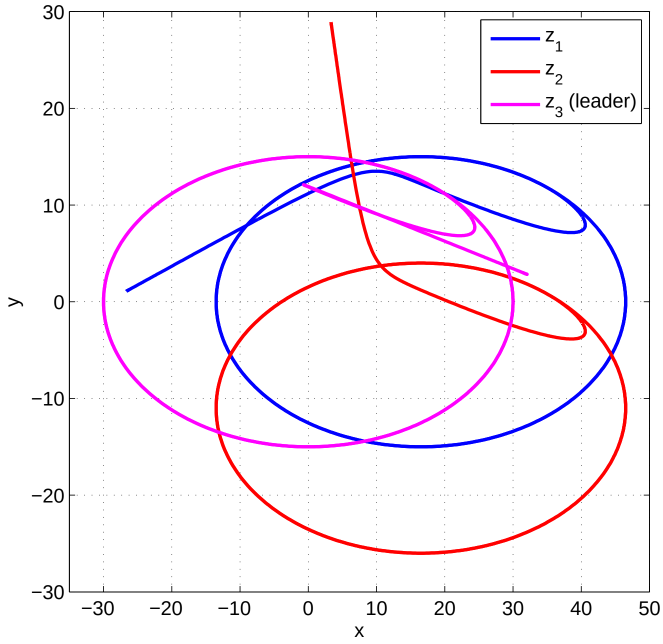

Fig. 2. Trajectories for a marching control for a triangular formation with out saturation

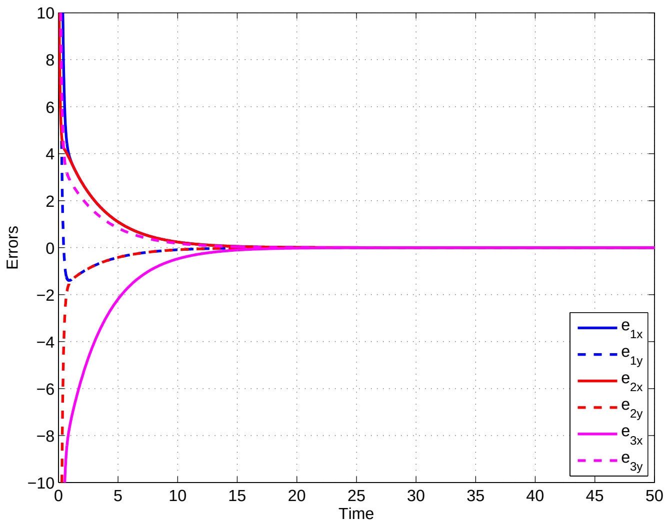

Fig. 3. Tracking error for a triangular formation with out saturation

the turning angle β_i, it can be seen in Figure 2 which converges to the desired trajectory asymptotically (Figure 3). Since there are no saturations on the turning angle of the vehicles, the control signal can become arbitrarily large and the turning angle can rotate without restrictions (Figures 4 and 5). Figure 6 shows the position and orientation of the robots at the time instant $t = 250$ recorded by the virtual reality environment

The second simulation is the marching control with trajectory given also by (21) with a triangular formation and saturation for the vehicles' turning angles as can be seen in Figure 7. There is converges to the desired trajectory (Figure 8). From the beginning of the simulation until time 93 there is saturation in vehicles' the turning angles. After time 93 the control stays in the the linear region (Figures 9 and 10). Figure 11 shows the

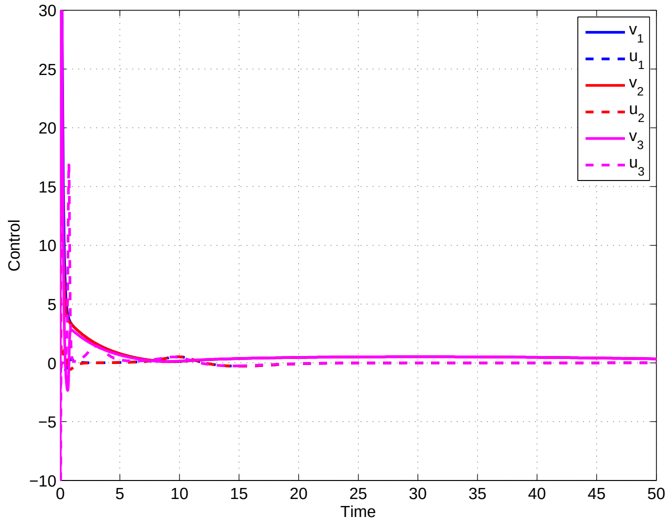

Fig. 4. Control signal for a triangular formation with out saturation

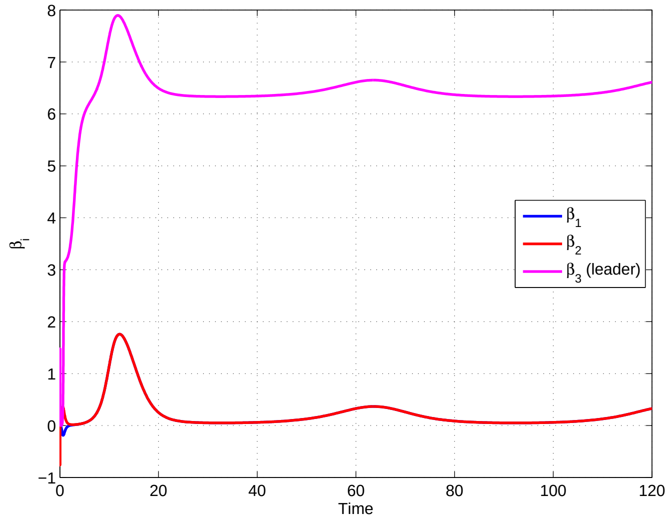

Fig. 5. Turning angle for the vehicles with out saturation

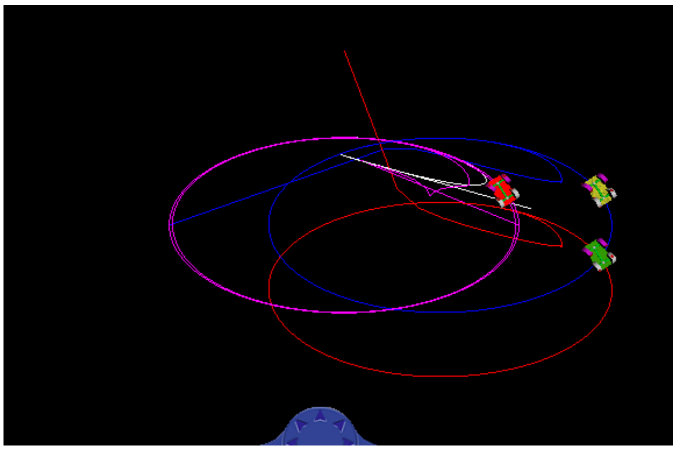

Fig. 6. Virtual reality simulation for a triangular formation with out saturation

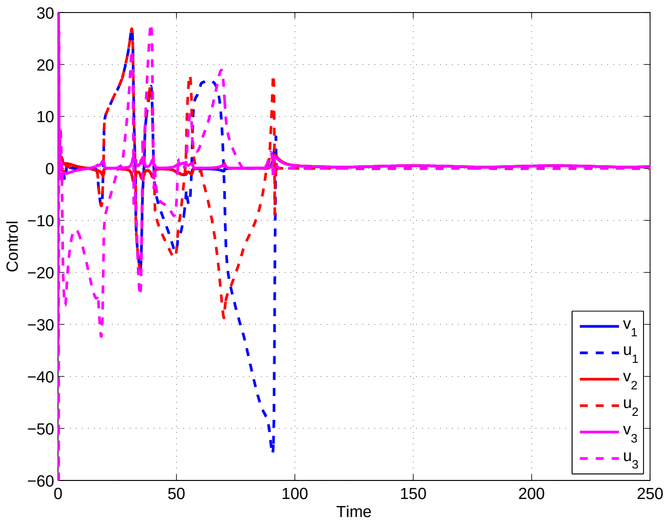

Fig. 9. Control signal for a triangular formation

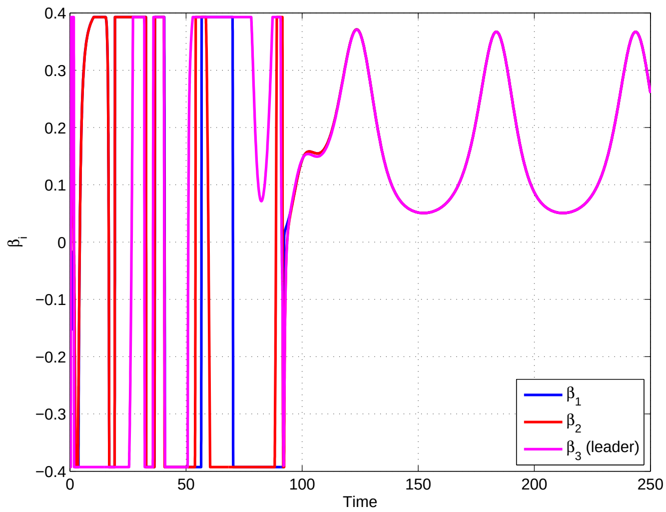

Fig. 10. Turning angle for the vehicles

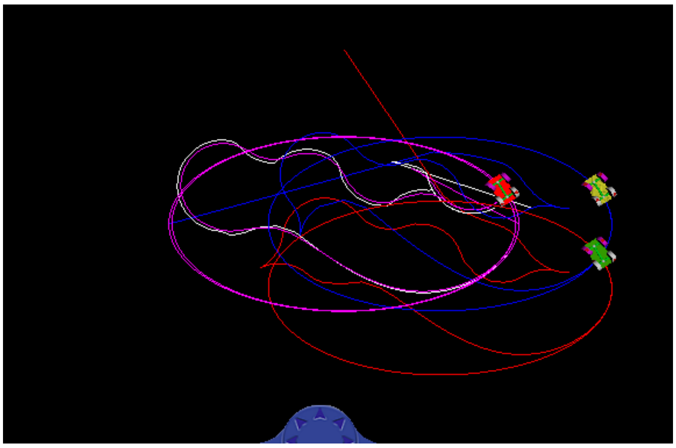

Fig. 11. Virtual reality simulation for a triangular formation

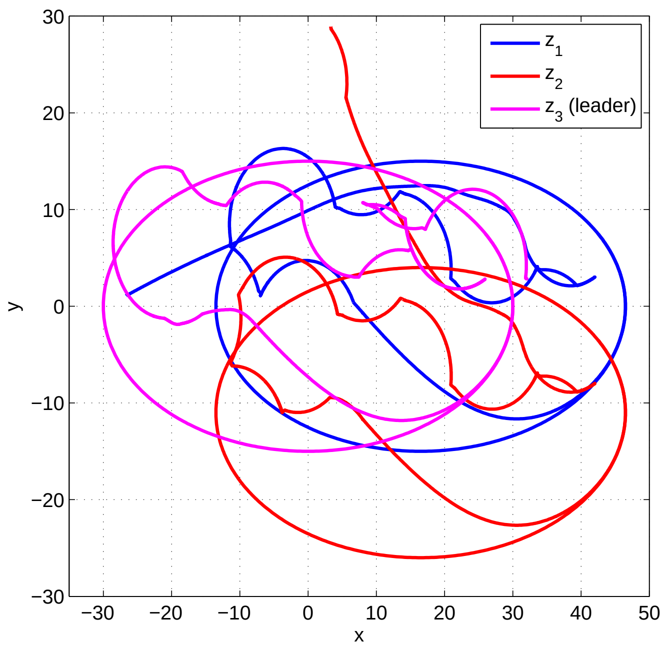

Fig. 7. Trajectories for a marching control for a triangular formation

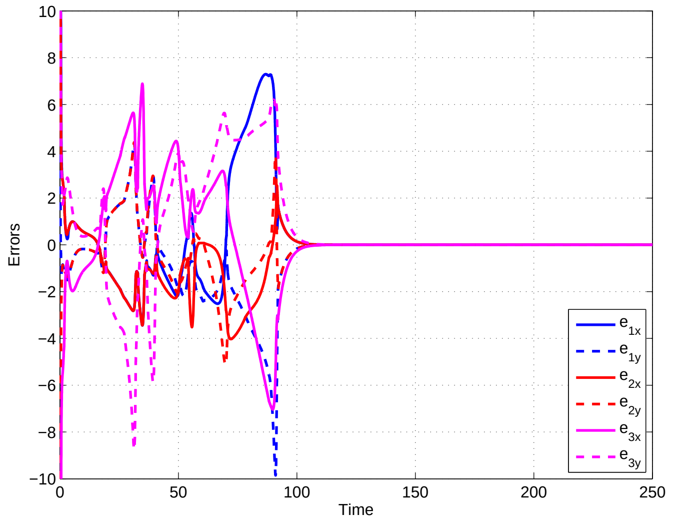

Fig. 8. Tracking error for a triangular formation

position of the mobile robots at the time instant $t = 250$ in the virtual reality environment.

5. CONCLUSIONS

This work presents a formulation for formation and marching control of car-like robots. The use of the point α is crucial to make the control laws to work avoiding the singularities in the kinematics of this type of robots. The convergence is guarantee when the robot are working in their linear region. In the case in which the front wheel control saturates, an anti-windup control is added. Simulations show in this case that the control laws still manages to converge to the linear region after some time. Future work would explore a theoretical determination of the region of the state space, formations and trajectories in which saturated control is able to keep the formation and marching of the robot as well as designed better control laws to solve these problems.

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