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Abstract

When the central bank minimizes a quadratic loss function depending upon the inflation gap and the output gap, a negative association between the variance of inflation and the variance of output emerges. The variance of output will be higher the greater is the preference of the central bank for stabilizing inflation. The use of certain ad-hoc interest rate rules analyzed in the literature may break the described negative relation. Central banks could reduce both variances. Data mainly from the US suggests that central banks use ad-hoc interest rate rules more than the minimization of the quadratic loss function.

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INTRODUCTION

In the old days there was a fierce discussion about the long run slope of the Phillips curve. The pioneering papers by Phillips (1958) and Samuelson and Solow (1960) proposed a possible permanent tradeoff between inflation and unemployment, or a long run positive relation between inflation and a measure of the output gap. In 1967 and 1968 Edmund Phelps (1967) and Milton Friedman (1968) assert that the long run tradeoff between inflation and unemployment was a myth. That would imply a permanent erosion of the real wage in the presence of inflation, something that could not last forever. The adjustment of wages would eliminate the possible short run tradeoff, sooner or later. Unemployment would converge to a value known, since then, as the natural rate.

In the following years, the school of rational expectations (Lucas (1972), Sargent and Wallace (1975), Barro (1976)) proposed almost the inexistence of the Phillips curve, or at least a very weak short run tradeoff between inflation and unemployment.

The traditional Keynesian school was almost debunked by the rational expectations ´s revolution (Lucas and Sargent (1979)). New Keynesians surged adopting optimization techniques and, in the majority of cases, maintaining the idea of Phelps and Friedman that there was a short run tradeoff between inflation and unemployment, but not a long run relation between these variables (see for example Mankiw and Romer (1991)).

In the new Keynesian perspective, several works have had an enormous impact. Taylor (1979) (1980) proposed that staggered prices and wages were compatible with rational expectations. In a similar approach, Calvo (1983) generated the very famous new Phillips curve, which has become a cornerstone of the new Keynesian perspective. Taylor (1993) proposed a rule for interest rates that was necessary for sticky price models to be stable or to avoid multiple solutions.

The new Phillips curve, coped with the Taylor rule proposed by Taylor himself in 1993, produces an interesting result where the long run tradeoff between inflation and unemployment is absent, but where there is a long run tradeoff between the variance of inflation and the variance of output. This relation was first proposed by Taylor (1979a) and then by other authors (see for example Taylor (1994), Fuhrer (1997), Bean (1998), Erceg, Henderson and Levin (1998), Cecchetti and Ehrman (1999) and Lee (1999)). What is the importance of this result?

The result is important because of its relation to economic policy: if a central bank is very concentrated in maintaining a stable inflation, there will be a higher variance of output; on the contrary, if the central bank is committed to stabilize output, there will be higher variance of inflation. There must be a balance between these two variances.

Under the “classical” assumptions of the new Keynesian school, central banks whose mandate is to maintain inflation in a target may generate high volatility of output, and for that reason also in employment and the real rate of interest. Is that convenient for society?

While answering this question is without doubt important, there is another question before. Is the tradeoff between the variance of inflation and the variance of output a fact in real life? Under what conditions? Do these conditions hold in reality? Taylor (1994) questions his own result at empirical level. The relation maybe does not hold because of the use of inefficient policies, but he does not provide examples.

These are the questions that this paper pretends to answer.

The paper is divided in three sections. The first one shows the “classical” new Keynesian approach, where the variance of inflation and the variance of output show an inverse relation. Second section takes the same new Phillips curve of the first section but assumes an ad-hoc monetary policy in the spirit of Romer´s 2000 paper (Romer (2000)). If that is the case, the relation between the variance of inflation and the variance of output may have different forms. Finally, section three shows some empirical evidence, which suggests that the conditions that must hold for having a permanent tradeoff between the variance of inflation and the variance of output are absent in the US, and probably in all the other countries of the G7.

I.-THE NEW KEYNESIAN "CLASSICAL" APPROACH: THERE IS A TRADEOFF BETWEEN THE VARIANCE OF INFLATION AND THE VARIANCE OF OUTPUT

In the simplest new Keynesian model prices are staggered (Taylor (1980), Calvo (1983)), which generate the new Phillips curve. There is an old IS curve linking output and the real rate of interest negatively and the central bank minimizes a loss function. This one depends upon the quadratic deviations of inflation from its target (the inflation gap or the inflation cycle) and output from its normal or potential level (the output gap).

The simplest new Phillips curve is the one derived by Mankiw and Reis (2001) based in the work by Calvo (1983), which may be described as:

$$\pi_t = E_t\pi_{t+1} + \delta(y_t - y^*) + e_t \quad (1)$$

Where π_t is the actual rate of inflation; $E_t\pi_{t+1}$ is the conditional expected value of inflation in the next period. The variable y_t is present output and y^* is potential output. Therefore y_t-y^* is the output gap. The term e_t is an independent uncorrelated normally distributed supply random shock with zero mean and constant variance σ^2.

The IS curve in the economy is defined as.

$$y_t = H - br_t + v_t \quad (2)$$

This function establishes a consistent negative relation between income (y) and the real rate of interest r. Parameter v is an independent uncorrelated random shock normally distributed with zero mean and variance equal to σ_v^2.

Parameter H could represent a term related to fiscal policy parameters, where increases in government expenditure increases H (dH/dG>0) or increases in the income tax rate reduces H (dH/dt<0). Other variables from the private sector, like autonomous expenditure, may be also in the parameter H of the traditional IS curve.

In the new Keynesian "classical" approach, the central bank has a well-defined objective, which in many cases consist of minimizing the following loss function

$$L_t = \varphi(\pi_t - \pi^*)^2 + (1-\varphi)(y_t - y^*)^2 \quad (3)$$

Where π^* is the targeted inflation. $(\pi_t - \pi^*)$ is the inflation gap or the inflation cycle.

The loss function is quadratic as in the Barro-Gordon approach (Barro and Gordon (1983)).[1] The best possible outcome for the central bank is to have actual inflation π_t equal to the inflation target π^* and present output y_t equal to natural output y^*. Every other possibility produces a positive loss function.

The parameter φ is a representation of how important is for the central bank the stability of inflation around its target in comparison to the stability of output ($0 < \varphi < 1$). When φ is 1 the central bank is just committed to set inflation in its target. On the contrary, when $\varphi=0$ ($1-\varphi=1$) the central bank just cares about the stability of output. In practice the central bank cares about the two objectives.

The primal problem to solve is to minimize equation (3) of the loss function subject to equation (1) of the Phillips curve. That will generate a relation between the output gap $(y_t - y^*)$ and a version of the inflation gap $(\pi_t - \pi^*)$. To get this function the central bank has to set a monetary rule for the rate of interest, which is the very well-known Taylor rule (see Taylor (1993)).

Minimization of (3) subject to (1) gives, as a result:

$$y_t - y^* = -\frac{\varphi\delta(E_t\pi_{t+1} - \pi^*)}{(\varphi\delta^2 + (1-\varphi))} - \frac{\varphi\delta e_t}{(\varphi\delta^2 + (1-\varphi))} \quad (4)$$

This equation, which we call the aggregate demand, shows a negative association between the output gap $(y_t - y^*)$ and a proxy of the inflation gap $(E_t\pi_{t+1} - \pi^*)$.

For equation (4) being in fact the aggregate demand, the central bank has to set a rule for interest rates. Substituting (4) in the IS curve (2) we get:

$$r_t = \frac{H - y^*}{b} + \frac{\varphi\delta(E_t\pi_{t+1} - \pi^*)}{(\varphi\delta^2 + (1-\varphi))b} + \frac{\varphi\delta e_t}{(\varphi\delta^2 + (1-\varphi))b} + \frac{v_t}{b} \quad (5)$$

[1] An important difference of this approach with the Barro-Gordon version is that in the last one the target for income is greater than the natural output y*.

Equation (5) is the famous Taylor rule in real interest rates (the MP equation of the Romer-Taylor model. See Romer (2000), Taylor (2000)). When inflation and its expectations are stable $E_t\pi_{t+1}=\pi^*$, and in absence of random shocks coming from supply and/or demand:

$$r_t = \frac{H-y^*}{b} = r^* \quad (6)$$

Where r^* is the natural rate of interest, a term coined more than one hundred years ago by the Swedish economist Knut Wicksell (Wicksell (1898)).

In the traditional IS-LM analysis (Hicks (1937)), the natural rate of interest depends upon the parameter H, which instead is driven in part by fiscal policy. Higher government expenditure, or lower income tax rate, are factors that increase the natural rate of interest and then, through the Taylor rule (5), the actual real rate of interest.[2]

The dual problem consists of taking the Taylor rule (5) and substitute it in the IS curve (2). The result is the aggregate demand (4). Clearly, the Phillips curve (1) and the aggregate demand (4) are the relevant equations to solve for inflation (π_t) and output (y_t), while the IS curve solves for the real interest rate r_t.[3]

The way to solve for inflation and output is to substitute the output gap (y_t-y^*) of the aggregate demand equation (4) in the Phillips curve (1), which gives, as a result:

$$\pi_t = \frac{(1-\varphi)}{\varphi\delta^2+(1-\varphi)} E_t\pi_{t+1} + \frac{\varphi\delta^2}{\varphi\delta^2+(1-\varphi)} \pi^* + \frac{(1-\varphi)}{\varphi\delta^2+(1-\varphi)} e_t \quad (7)$$

Inflation depends upon its future expectations, the inflation target and the supply shock e_t. In absence of random shocks inflation would be a weighted average of its future expectation and its target.

To solve analytically the difference equation (7) we use the forward operator (see Hamilton (1994)).

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[2] A very different result surges when the IS comes from consumer optimization (see McCallum and Nelson (1999) or King (2000)) in this case the natural rate of interest is the subjective rate of discount of the utility of consumers.

[3] It is interesting to note that the form of the IS curve is irrelevant in the solution of inflation and output. We could be using a new IS curve, like the one used in McCallum and Nelson (1999), King (2000) and Blanchard(2008), and the results for inflation and output and their variances would be exactly the same.

$$E_t\pi_{t+1} = L^{-x}\pi_t \qquad (8)$$

Also, to simplify notation, we call

$$j = \frac{\varphi\delta^2}{(\varphi\delta^2+(1-\varphi))} \quad (9)$$

$$1 - j = \frac{(1-\varphi)}{(\varphi\delta^2+(1-\varphi))} \quad (10)$$

Solution for inflation is then

$$\pi_t = \pi^* + (1-j)\sum_{i=0}^{\infty}(1-j)^i E_t e_{t+i} \quad (11)$$

But

$$\sum_{i=0}^{\infty}(1-j)^i E_t e_{t+i} = e_t \quad (12)$$

Since $E_t e_{t+i}$=0 for $i > 0$

Then the definite reduced form for inflation is

$$\pi_t = \pi^* + (1-j)e_t \quad (13)$$

At the same time, to solve for y it is possible to set aggregate demand (4) as:

$$y_t - y^* = -\frac{j}{\delta}(E_t\pi_{t+1} - \pi^*) - \frac{j}{\delta}e_t \qquad (14)$$

But using (13) it is possible to show that:

$$E_t\pi_{t+1} = \pi^* \quad (15)$$

Therefore, the reduced form for output is:

$$y_t = y^* - \frac{j}{\delta}e_t \qquad (16)$$

Positive supply shocks (e_t<0) generate a situation where output rises above the natural level and inflation falls. However, given the assumptions of the model the impact of these shocks dilutes in just one period.

We will show that there is a tradeoff between the variance of output and the variance of inflation. Output will be more variant the more worried is the central bank maintaining inflation around the target π^* and vice versa.

The non-conditional expectation for inflation in equation (13) is π^*, since the non-conditional expectation for the disturbance term e_t is zero. Therefore, the variance of inflation is:

$$Var(\pi_t) = E(\pi_t - E(\pi_t))^2 = E(\pi_t - \pi^*)^2 = (1-j)^2\sigma^2 \quad (17)$$

At the same time, the non-conditional expectation for output in (16) is y^* because of the same reason that in the previous equation. Then

$$Var(y_t) = E(y_t - E(y_t))^2 = E(y_t - y^*)^2 = \frac{j^2}{\delta^2}\sigma^2 \quad (18)$$

Parameter j is directly related to φ, which instead represents how much the central bank cares about the variance of inflation compared with the variance of output. When φ=1, then j=1 and the central bank cares just about maintaining inflation in its target π^*. When φ=0, j=0 and the central bank cares just about to maintain output at its natural level y^*.

There is a monotonic relation between j and φ, which can be seen taking the derivative of j with respect to φ in (9):

$$\frac{dj}{d\varphi} = \frac{\delta^2}{(\varphi\delta^2 + (1-\varphi))^2} > 0 \quad (19)$$

$$\frac{dVar\,(\pi_t)}{dj} = -2(1-j)\sigma^2 < 0 \quad (20)$$

$$\frac{dVar(y_t)}{dj} = 2\frac{j}{\delta^2}\sigma^2 > 0 \quad (21)$$

The more the central bank cares to maintain inflation in its target π^* (the higher are φ and j), the lower is the variance of inflation and the higher is the variance of output. There is a tradeoff between the variance of inflation and the variance of output.

The tradeoff between the variance of inflation and the variance of output is a topic already analyzed by different authors, like Taylor (1979a) (1994), Fuhrer (1997), Bean (1998), Erceg, Henderson and Levin (2000).

The highest possible variance of output and the lowest possible variance for inflation occur when φ=1=j, in which case:

$$Var(\pi_t) = 0 \quad (22)$$

$$Var(y_t) = \frac{\sigma^2}{\delta^2} \quad (23)$$

The highest possible variance of inflation and the lowest possible variance for output occur when φ=0=j, when

$$Var(\pi_t) = \sigma^2 \quad (24)$$

$$Var(y_t) = 0 \quad (25)$$

The variance´s function is a relation between the variance of inflation and the variance of output, which is found solving for j in (18) and substituting in (17), which gives, as a result:

$$Var(\pi_t) = \sigma^2 - 2\big(Var(y_t)\big)^{1/2}\delta\sigma + Var(y_t)\delta^2 \quad (26)$$

It is possible to prove that this function has a negative slope, since the first derivative of the variance of inflation with respect to the variance of output is:

$$\frac{d(Var(\pi_t))}{d(Var(y_t))} = -\big(Var(y_t)\big)^{-\frac{1}{2}}\delta\sigma + \delta^2 \quad (27)$$

For this derivate being negative, it is necessary to have:

$$-\big(Var(y_t)\big)^{-\frac{1}{2}}\delta\sigma + \delta^2 < 0 \quad (28)$$

But that means:

$$\frac{\sigma^2}{\delta^2} > Var(y_t) \quad (29)$$

This is true because the maximum variance of y is exactly σ^2/δ^2.

At the same time, the second derivative is positive, showing that there is a convex relation in the plane where the variance of inflation is in the vertical axis and the variance of output is in the horizontal axis.

This can be seen taking the second derivative in (27).

$$\frac{d^2Var(\pi_t)}{d(Var(y_t))} = \frac{1}{2}(Var(y_t))^{-3/2}\delta\sigma > 0 \qquad (30)$$

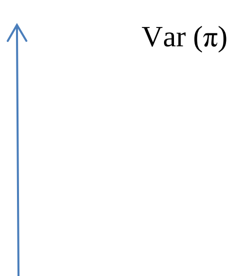
Which implies that in the plane where the variance of inflation is in the vertical axis, and the variance of output is in the horizontal axis, the relation between these two variables is inverse and convex to the origin.

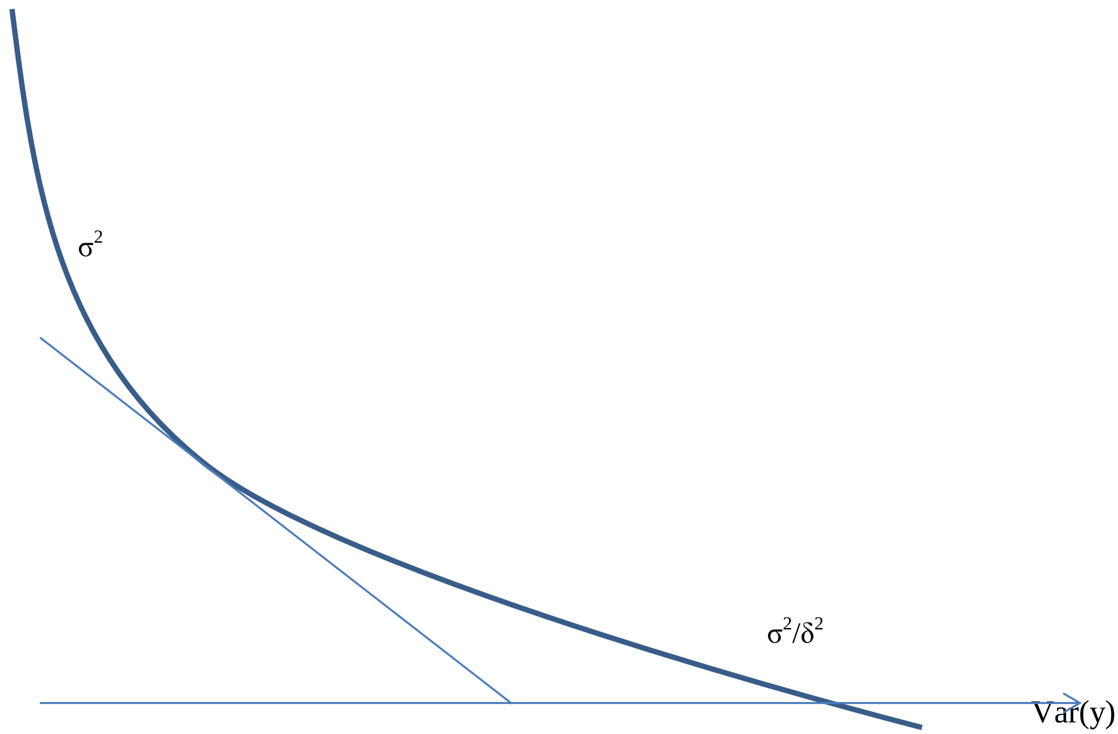
On the other hand, the expected value of the loss function (3) is a linear relation between the variance of inflation and the variance of output, which may be seen applying the non-conditional expectation's operator to equation (3) and considering that the non-conditional expectation for inflation and output are π^* and y^*, respectively.

$$E(L_t) = \varphi E(\pi_t - \pi^*)^2 + (1-\varphi)E(y_t - y^*)^2 = \varphi E(\pi_t - E(\pi_t))^2 + (1-\varphi)E(y_t - E(y_t))^2 = \varphi Var(\pi_t) + (1-\varphi)Var(y_t) \quad (31)$$

The expected result for the variance of inflation and the variance of output occurs in the tangency between the variance's function (26) and the expected loss function (31). While there is a unique variance's function, there is a dense map of expected loss functions, all of them linear.

Figure 1: The tradeoff between the variance of inflation and the variance of output in the "classical" new Keynesian model.





II.- AD HOC MONETARY AND FISCAL POLICES: IS THERE STILL A TRADEOFF BETWEEN THE VARIANCE OF INFLATION AND THE VARIANCE OF OUTPUT?

Do central banks optimize? Not necessarily. Information is not easy to obtain. To optimize central banks may have very good calculations of potential output as well as the parameters of the Phillips curve. There must be also a good agreement among the people compounding the board of governors of the bank. Optimization in these conditions is difficult to reach because there may be different estimations of the Phillips curve, or the confidence intervals are so large that there is a lot of uncertainty. There are simple stabilizing rules that may substitute optimization and to reduce costs of acquiring information.

A derivation of the very simple model proposed by Romer (2000) and Taylor (2000) and known as the Romer-Taylor model (Koenig (2008)) may be a good theoretical framework to compare with the optimizing model already analyzed in the previous section.

The proposed model would have again staggered prices (Koenig (2008)), the old IS curve and a simple ad-hoc Taylor rule a la Romer (2000), where there is a direct positive link between the real rate of interest and inflation (the MP function). The existence of staggered contracts would provide, as a result, the new Phillips curve.

The complete model may be then described as:

$$\pi_t = E_t\pi_{t+1} + \delta(y_t - y^*) + e_t \quad (32)$$

$$y_t = H - br_t + v_t \quad (33)$$

$$r_t = \Omega\pi_t \quad (34)$$

Where equation (32) is the new Phillips curve, which is identical to equation (1). (33) is the IS curve, again identical to equation (2) and (34) is a very simple Taylor rule in the Romer (2000) spirit. The central bank wants to stabilize inflation and then sets a rule where the nominal interest rate r+π is over indexed to the observed inflation rate (The Taylor (1993) principle). Intuitively, the more the central bank wants to stabilize inflation, the higher is the parameter Ω.

The combination of equations (33) and (34) generates the aggregate demand:

$$y_t = H - b\Omega\pi_t + v_t \quad (35)$$

In this demand actual output is related negatively to present inflation.

The solution for inflation and output is given by the Phillips curve (32) and aggregate demand (35). Substituting (35) in (32) we get:

$$\pi_t = hE_t\pi_{t+1} + \delta h(H - y^*) + \delta hv_t + he_t \quad (36)$$

Where

$$h = \frac{1}{1+b\Omega\delta} \quad (37)$$

Using the forward operator, it is possible to solve for the stochastic difference equation (36)

$$E_t\pi_{t+1} = L^{-1}\pi_t \quad (38)$$

Solution for inflation is:

$$\pi_t = \frac{\delta h(H-y^*)}{(1-h)} + \delta h \sum_{i=0}^{\infty} h^i E_t v_{t+i} + h \sum_{i=0}^{\infty} h^i E_t e_{t+i} \quad (39)$$

But since $E_t v_{t+i} = E_t e_{t+1} = 0$ for i>0 and substituting again h from (37)

$$\pi_t = \frac{(H-y^*)}{b\Omega} + \frac{\delta v_t}{(1+b\Omega\delta)} + \frac{e_t}{(1+b\Omega\delta)} \quad (40)$$

Which in turn implies, for output

$$y_t = y^* + \frac{v_t}{(1+b\Omega\delta)} - \frac{b\Omega e_t}{(1+b\Omega\delta)} \quad (41)$$

(40) and (41) are reduced forms for inflation and output. Inflation is determined by fiscal policy parameters (H), monetary policy parameters (Ω) and demand and supply shocks, being v demand shocks and e supply shocks.

Output is determined by demand and supply shocks but not by structural fiscal policy parameters.

Positive demand shocks (v>0) generate higher inflation and higher output but just by one period. Positive supply shocks (e<0) produce higher output and lower inflation also only for one period. Higher potential output raises output in a one to one basis and in a permanent way and reduces inflation also permanently.

In the "classical" new Keynesian solution the Taylor rule was designed in such a way that all parameters of the IS curve were absent of the solution for inflation and output. Instead, in this case demand matters in the solution of inflation and demand shocks impact output temporarily.

The non-conditional expectation for inflation and output is:

$$E(\pi_t) = \frac{H-y^*}{b\Omega} = \frac{r^*}{\Omega} \quad (42)$$

$$E(y_t) = y^* \quad (43)$$

And then the variance for both variables may be described as:

$Var(\pi_t) = E(\pi_t - E(\pi_t))^2 = \frac{(\delta^2\sigma_v^2+\sigma^2)}{(1+b\Omega\delta)^2}$ (44)

$Var(y_t) = E(y_t - E(y_t))^2 = \frac{(\sigma_v^2+b^2\Omega^2\sigma^2)}{(1+b\Omega\delta)^2}$ (45)

The ratio between these two variances is:

$\frac{Var(\pi_t)}{Var(y_t)} = \frac{\delta^2\sigma_v^2+\sigma^2}{\sigma_v^2+b^2\Omega^2\sigma^2}$ (46)

It is very clear from equations (44) to (46) that the higher is Ω, or the tougher is monetary policy, the lower is the variance of inflation and the lower is the ratio between the variance of inflation and the variance of output. That is to say, higher ex-ante interest rates reduce the absolute variance of inflation and reduce also the variance of inflation when compared with the variance of output, but what about the absolute variance of output?

Taking the derivative of the variance of output with respect to Ω in (46) and after tedious algebra:

$\frac{dVar(y_t)}{d\Omega} = \frac{2[(b^2\Omega+b^3\Omega^2\delta)\sigma^2-(b\delta+b^2\Omega\delta^2)\sigma_v^2]}{(1+b\Omega\delta)^2} \geq\leq 0$ (47)

The response of the absolute variance of output to tougher monetary policy (higher preoccupation of the central bank about inflation) is ambiguous. If there were only supply shocks, then higher Ω would generate higher variance of output and lower variance of inflation as in the "classical" new Keynesian solution. Instead, if supply shocks were absent, a tougher monetary policy would generate lower variance for both output and inflation. The long run tradeoff between the variance of output and the variance of inflation does not necessarily hold under this ad-hoc Taylor rule.

When Ω is equal to zero, the variance of inflation and the variance of output are

$Var(\pi_t) = \delta^2\sigma_v^2 + \sigma^2$ (48)

$Var(Y_t) = \sigma_v^2$ (49)

Instead, when Ω tends to infinity both variances take the following results:

$Var(\pi_t) = 0$ (50)

$Var(y_t) = \frac{\sigma^2}{\delta^2}$ (51)

Clearly the frontier between the variance of inflation and the variance of output in this case is to the right of the same frontier when the central bank minimizes the loss function. The extreme where Ω=0 shows higher variance for both, inflation and output, that in the case where the central bank optimizes (φ=0). The extreme where Ω tends to infinity shows the same variances for inflation and output that when the central bank optimizes (φ=1). The case where the bank optimizes is one producing an efficient frontier, while if the bank does not optimize the obtained frontier is not efficient and must be to the right of the efficient frontier.

Some simulations show different frontiers when Ω is varying and under different assumptions for the value of the other parameters.

We consider three scenarios: in all of them b=0.5 and δ=1. In the first one $\sigma^2=\sigma^2_v=1$. In the second one the variance of the disturbance term of the IS function is much greater than the variance of the same concept in the Phillips curve: $\sigma^2_v=5$; $\sigma^2=0.3$. The third scenario shows the other extreme case, where the variance of the disturbance term of the Phillips curve is much greater than the variance of the error term in the IS curve: $\sigma^2=5$; $\sigma^2_v=0.3$.

Simulations were performed with 1000 constructed observations, where Ω varies 0.1 points going from zero to 99.9. Results of the three scenarios may be seen in figures 2, 3 and 4.

Figure 2: Frontier between the variance of inflation and the variance of output when $\sigma^2=\sigma^2_v=1$. (vertical axis shows the variance of inflation, horizontal axis shows the variance of output):

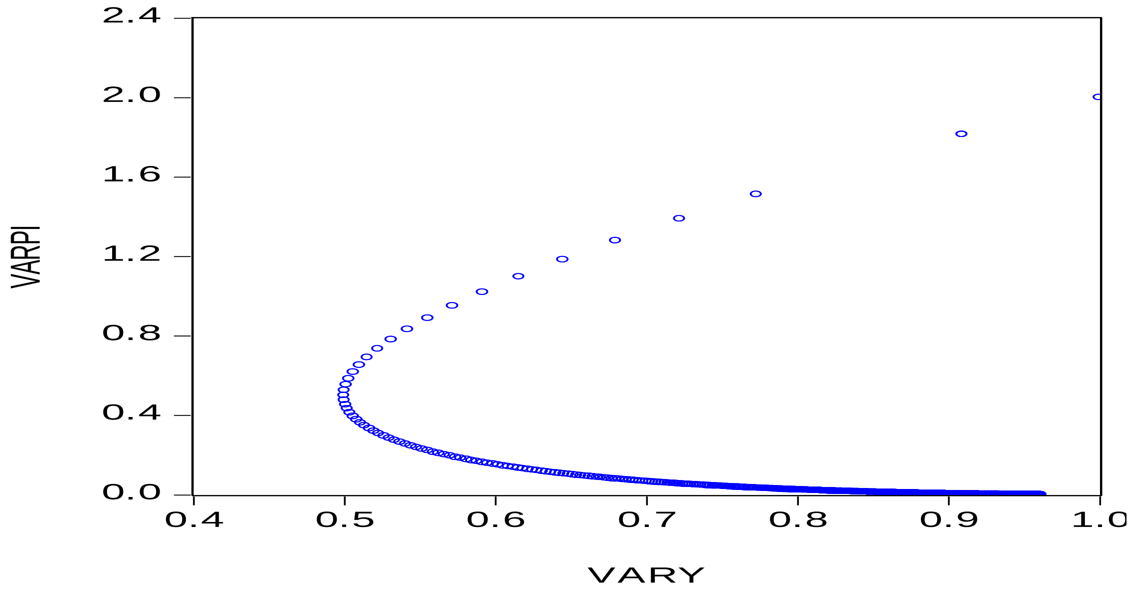


Figure 3: Frontier between the variance of inflation and the variance of output when σ^2_v=5 and σ^2=0.3 (vertical axis shows the variance of inflation; horizontal axis shows the variance of output):

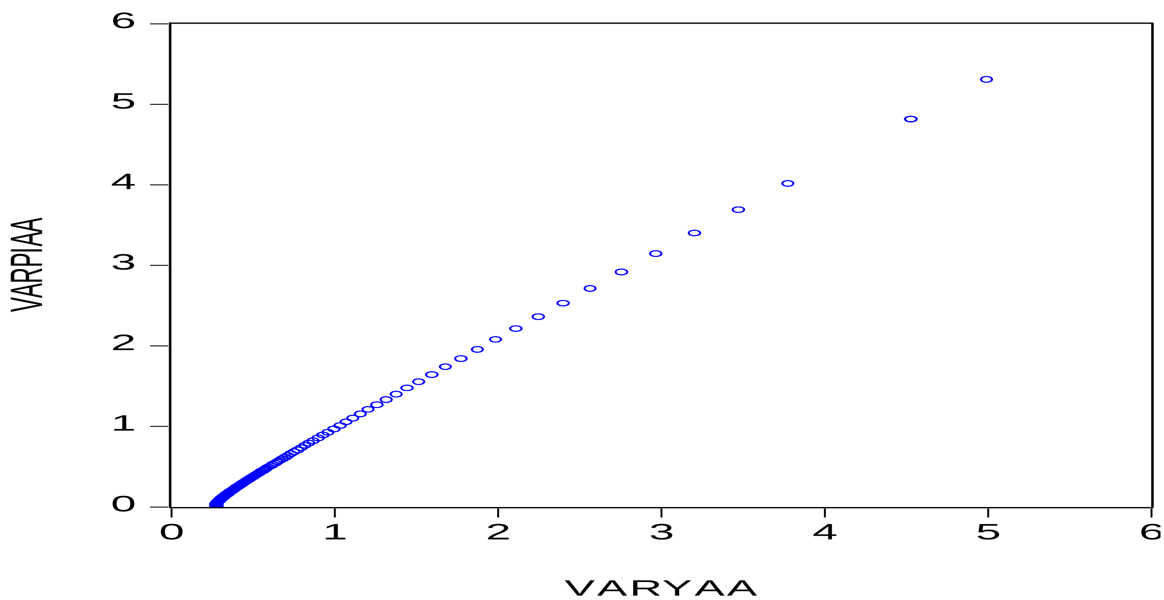
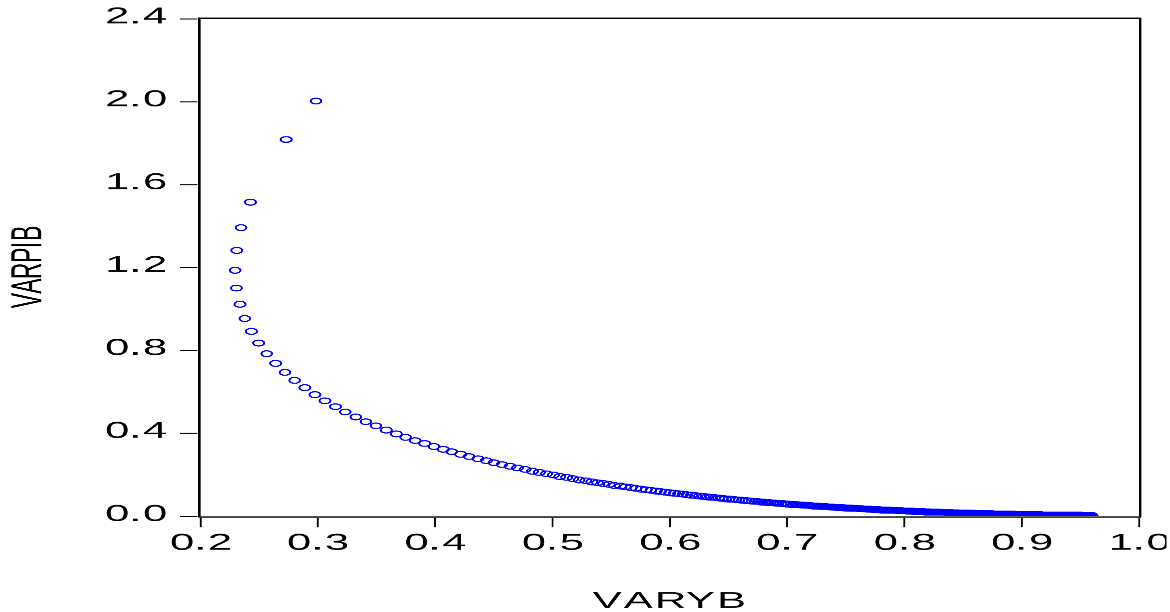


Figure 4: Frontier between the variance of inflation and the variance of output when the σ^2=5 and σ^2_v=0.3 (vertical axis shows the variance of inflation; horizontal axis shows the variance of output):



These figures show that there is not necessarily a monotonic relation between the variance of inflation and the variance of output. When the variance of the disturbance term of the IS curve ($\sigma^2{}_v$) is much greater than the variance of the disturbance term of the Phillips curve (σ^2) (Figure 3), there is a perverse relation where the variance of inflation and the variance of output are positively correlated. The other extreme case, where σ^2 is much greater than $\sigma^2{}_v$, shows almost a continuous tradeoff between the variance of inflation and the variance or output. When $\sigma^2{}_v=\sigma^2$ there is almost a perfect parabola relating the variance of inflation with the variance of output.

All of these frontiers are inefficient with respect to the hypothetical frontier where under the same value of the parameters ($\sigma^2{}_v$, σ^2, δ and b), and varying the parameter φ from zero to one, the central bank minimizes the loss function. That is to say these frontiers are to the left of the hypothetical more efficient frontier seen in section I (figure 1).

III.- DO CENTRAL BANKS MINIMIZE A LOSS FUNCTION?

The second section of this paper could be criticized on the grounds that if the central bank is rational it should follow a program like the one explained in section I, reaching a situation where the most efficient tradeoff between the variance of inflation and the variance of output surges.

Some relatively casual look to data in different countries suggests that after all central banks do not act as optimizer agents and that perhaps they follow more ad-hoc rules for interest rates.

Minimizing a loss function like the one in equation (3) subject to a Phillips curve, as in equation (1), should give, as a result, an aggregate demand where there is a negative relation between the output gap and some kind of inflation gap. Is there this kind of relation in developed countries, namely those members of the G7?

We get a correlation between the output gap published in the World Economic Outlook (WEO) database of the International Monetary Fund (IMF) and a present and future inflation gap (one year ahead), obtained with data from the same source from 1980 to 2012, and generated through a Hodrick-Prescott filter. In none case there is a negative association and in many cases there is a positive significant relation (see table 1). That observation suggests that probably the aggregate demand has a different form than the one obtained by an optimizing behavior and then probably central banks do not optimize.

Table 1: Correlation between the actual output gap and present and future inflation gap in G7 countries (1980 2012 annual terms).

	Present inflation gap	Future inflation gap
Canada	0.25	0.59

France	0.29	0.34
Germany	0.45	0.41
Italy	0.31	0.21
Japan	0.26	0.62
United Kingdom	0.09	0.43
United States	0.29	0.46

Source: Elaborated by the author with data from the World Economic Outlook database (International Monetary Fund). October 2013.

Another interesting observation is the one that surges when we correlate the real interest rate in the US to inflation or to a measure of the inflation gap. We calculate a real rate ex post with annual data from the Board of Governors of the Federal Reserve Bank of the rate of the Federal Funds and the observed inflation of the next year, in the following way:

$$r_t = \frac{R_t - \pi_{t+1}}{(1+\pi_{t+1})} \quad (52)$$

Where r is the real rate of interest of the year t; R_t is the nominal interest rate of federal funds in year t; π_{t+1} is inflation in year t+1.

Theoretically, if the central bank were an optimizing agent, the real rate of interest should be correlated positively with the future inflation gap. Instead, if the central bank followed a more ad-hoc policy, maybe the real interest rate should be more correlated with the level of inflation.

We correlate the calculated ex-post real interest rate with the present inflation gap, the future inflation gap (one year ahead), the contemporaneous level of inflation and the future level of inflation (one year ahead). Results are the following:

Table 2: Correlation of the real rate of federal funds in the US with different variables related to inflation

	Correlation of the real rate with variables
Contemporaneous inflation gap	0.027

Future inflation gap	-0.691
Contemporaneous level of inflation	0.263
Future level of inflation	-0.191

Source: Elaborated by the author with data from the World Economic Outlook Database October 2013 and Board of Governors of the Federal Reserve Bank.

The higher correlation of the real rate of interest of federal funds is with the future inflation gap. However, this relation is negative when theoretically it should be positive. The way in which the real rate was constructed is related to this correlation.

The correlation between inflation and the current inflation gap is almost zero. Instead, the correlation between inflation and the contemporaneous level of inflation, without being very high, could be significant and with the correct sign.

We perform a last exercise where we run different regressions trying to capture a Taylor rule in the US. Again, we use the definition of the real interest rate in equation (52). It is necessary to consider that the monetary policy of the last years in the US has been quite particular, with an almost fixed nominal interest rate near the zero level. For this reason we perform annual regressions from 1980 or 1981 to 2007. The regressions take the following form:

$$r_t = A_0 + A_1 I\pi_t + A_2(y_t - y_t^*) + A_3 r_{t-1} + f_t \quad (53)$$

Where r is the real rate of interest; Iπ is an indicator of some level of inflation, or an indicator or some inflation cycle; y_t-y^* is the output gap and f_t is an error term normally distributed with zero mean. The possibility of a partial adjustment is considered and for that reason we include the lag of the real interest rate.

The value Iπ is defined in six different ways:

π_t : The contemporaneous level inflation

π_{t-1}: The lag of the level inflation

π_{t+1} : The forward value of the level inflation

$\pi_t - \pi_t^*$: The contemporaneous inflation gap

$\pi_{t-1} - \pi_{t-1}^*$: The lag of the inflation gap

$\pi_{t+1} - \pi_{t+1}^*$: The forward value of the inflation gap

Results for these regressions are:

Table 3: Regression

Dependent variable: real interest rate of the current year

Annual information from 1980 to 2007

Method of estimation: Generalized method of moments (GMM)

Indicator of inflation

	π_t	π_{t-1}	π_{t+1}	$\pi_t-\pi_t^*$	$\pi_{t-1}-\pi_{t-1}^*$	$\pi_{t+1}-\pi_{t+1}^*$
Constant	-0.88 (-1.8)	-0.4 (-1.7)	0.19 (0.3)	-0.01 (-0.09)	0.15 (1.1)	0.62 (0.9)
Indicator of inflation	0.33 (2.2)	0.21 (3.0)	0.02 (0.13)	0.38 (1.9)	0.20 (2.3)	-0.87 (-1.0)
Output gap	0.04 (0.8)	0.08 (1.4)	0.02 (0.4)	-0.12 (-2.1)	-0.03 (-0.6)	0.16 (0.7)
Lag of the real interest rate	0.65 (7.2)	0.45 (4.4)	0.51 (4.1)	0.87 (4.5)	0.64 (5.5)	-0.04 (-0.06)
R^2	0.57	0.51	0.34	0.37	0.28	0.25
D.W.	1.36	1.76	1.7	1.44	1.80	0.55
J.B	1.9	1.4	1.0	1.3	1.1	2.8
Q(12)	24.9	13.0	8.9	20.1	10.0	8.1
J	0.84	1.2	1.5	0.5	1.8	3.4
$X^2(3)$	1.3	2.16	2.9	0.78	3.05	2.13

R^2: Coefficient of determination; D.W.: Durbin-Watson statistic; J.B. Jarque-Bera statistic; J: J statistic; Q(12): Box-Pierce statistic of the correlogra;. $X^2(3)$: Sargan´s instrumental validity test. Instruments for regressions: For columns: regression 1: $\pi_{t-1}-\pi_{t-1}^*$; $\pi_{t-2}-\pi_{t-2}^*$; r_{t-1}; r_{t-2}; $y_{t-1}-y_{t-1}^*$; π_{t-1}. For regression 2: π_{t-1}; $\pi_{t-2;}$ r_{t-1}; $r_{t-2;}$ $y_{t-1}-y_{t-1}^*$; For regression 3: r_{t-1}; $r_{t-2;}$ r_{t-3}; π_{t-1}; $y_{t-1}-y_{t-1}^*$. For regressions 4 and 5: π_{t-1}; $\pi_{t-2;}$ r_{t-1}; $r_{t-2;}$ $y_{t-1}-y_{t-1}^*$. For regression 6: $\pi_{t-1;}$ $\pi_{t-1}-\pi_{t-1}^*$; $\pi_{t-2}-\pi_{t-2}^*$; r_{t-1}; $y_{t-1}-y_{t-1}^*$

Source: Elaborated by the author with data from interest rates of the Board of Governors of the Federal Reserve Bank; inflation and output gap from the World Economic Outlook database, October 2013.

The best adjustment in terms of the R^2 statistic is the one where the indicator of inflation is the actual level of inflation. In general, equations considering levels of inflation (contemporaneous, past and future) have better adjustment than those considering the inflation cycle as a proxy of the difference between actual inflation and targeted inflation. That contradicts the “classical” new Keynesian approach, where the Taylor rule is more related to the inflation gap than to the level of inflation.

The past value of the real rate of interest is significant and positive in five out of the six cases. In those equations where the indicator of inflation is the level of that variable, it is possible to reject that the value of the coefficient of the lagged real interest rate is one, and in fact it has to be smaller. Since all estimations show values lower than one, models are stable. However, in the case where the level of inflation is the actual inflation gap, it is not possible to reject that the coefficient of the lagged real interest rate is different to one. In that situation the estimated equation could be unstable. [4]

We used a GMM estimation for two reasons: one because there could be a problem of endogeneity. A greater level of inflation (or the output gap) influences the real rate of interest, which at the same time perturbs also inflation (o the output gap). Instruments are different in almost all regressions because some of them (instruments) that were valid in some cases were not valid in others. The Sargan's instrumental validity test shows a good behavior to avoid the problem of endogeneity (see the last row of table 3 and Cuthberson, Hall y Taylor (1992. p 110) for an explanation of the test).

The other reason why we use GMM is because two equations involve future variables influencing present variables. In that case the typical estimation is performed through the GMM technique (see for instance Fuhrer and Rudebusch (2004)).

A surprising result is that the output gap is not significant in the majority of the cases. When it is significant it has the wrong sign (regression where the indicator of inflation is the contemporaneous inflation cycle). In the Taylor proposal (Taylor (1993)) the output gap was a determinant of the real interest rate. In the Romer model (Romer (2000)) the interest rate rule does not depend upon the output gap.

According to these results, the estimated Taylor rule is nearer to Romer´s style than Taylor´s proposal. Taylor´s proposal in 1993 coincides in an important way with the optimization problem seen in the first section of this paper, while Romer´s rule in 2000 coincides more with an ad-hoc rule. If there is an ad-hoc interest rate rule, it is not at all

[4] Tests for stationarity of the variables cannot conclude in a determinant way whether or not the variables involved in the Taylor equation are or not stationary. The partial adjustment model that we perform is an indirect way of checking for stability and, if the variables were integrated of order greater than zero, that stability would mean cointegration.

clear that there exist a long run tradeoff between the variance of inflation and the variance of output.

CONCLUSIONS

When the central bank minimizes a quadratic loss function depending on the output gap and the inflation gap, the resulting interest rate policy produces a tradeoff between the variance of output and the variance of inflation. Central banks committed to maintain inflation very near to a certain target will generate high volatility of output and vice versa. On average, inflation will be in the target and output will take its natural level. Economic policy may influence the variance of these variables, nonetheless.

Real observations challenge the hypothesis that central banks minimize a loss function. Interest rate rules seem to be more ad-hoc than generated by an optimization procedure. In that case, it is not clear that there is a tradeoff between the variance of inflation and the variance or output. There could be situations where both variances may increase or fall when monetary policy changes. Ideally, the central bank should minimize the correspondent loss function because in that case it will be in an efficient frontier, while in other cases both the variance of inflation and the one of output may be very high.

A problem in all this discussion is the nature of the proposed Phillips curve, which assumes always the natural rate hypothesis. If monetary and fiscal policies cannot change the long run level of output, then minimization of the proposed loss function makes sense. Nonetheless, there are different opinions asserting that there is not a natural rate of unemployment, at least for some values of low inflation, and that there is a long run tradeoff between inflation and unemployment (see for example Akerlof, Dickens and Perry (2000), Deveraux and Yeatman (2002), Graham and Snower (2002), Karanassou, Sala and Snower (2005)). If such tradeoff exists, the minimization function of the central bank probably does not make sense.

The reason for this last assertion is that if it is possible to increase inflation, and reduce unemployment in the same terms, the loss function making sense would be one where there is a preference for price stability, or probably for a targeted inflation, but there would be also a preference for higher output and there should not be a targeted output. The proposed function could be one where inflation is in quadratic terms while output in the targeted function could be linear and with a negative sign.[5] The central bank, as probably everybody, wants price stability and a higher output. In a world where there is a long run tradeoff between inflation and unemployment the optimal solution would imply some inflation with a bit less of unemployment and higher output.

What all of this discussion tries to say is that the preferences of the central bank are not independent of the existent long run tradeoff between inflation and unemployment. If the natural rate hypothesis prevails, the quadratic function set in this paper, and in many others, seems a consistent objective function of the central bank. However, if there is not a natural rate, the objective function should be other where the reduction of the long term unemployment matters. This is something that has to be explored in the near future.

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[5] A possible objective loss function would be the following:
$L_t = \varphi_1 \pi_t^2 - \varphi_2 y_t$

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