Revista Mexicana de Economía y Finanzas Nueva Época
Volumen 14 Primer Número Especial Aniversario, Agosto 2019, pp. 541-557
DOI: https://doi.org/10.21919/remef.v14i0.422

Limited Information and the Relation Between the Variance of Inflation and the Variance of Output in a New Keynesian Perspective

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(Primera recepción: 15/septiembre/2017, última recepción: 10/diciembre/2018 aceptado: 25/abril/2019)

Abstract

The objective of this paper is to analyze the effects on welfare of a monetary policy that establishes the reference interest rate at discrete intervals of time. The hypothesis is that because there is uncertainty about various disturbances that will occur in the period in which the referential interest rate is established, this can cause a loss of social welfare.
To analyze the problem, a model is proposed where the central bank minimizes a loss function. When there is perfect certainty, an efficient frontier between the variances of inflation and output is reached. With uncertainty the result is inefficient. This implies the need to discuss whether it would be convenient for the interest rate to be set contingently.
The main limitation of the work is perhaps that the model used makes a large number of abstractions, which allows it to be functional, but can leave out important aspects of reality. There seems to be very few papers, in any, that deal with the problem addressed in this work.
JEL Classification: E31, E32, E43, E52
Keywords: Inflation, variance of inflation, variance of output, interest rate's setting, inflation targeting

Información limitada y la relación entre las varianzas del producto y la inflación: análisis con el enfoque de la nueva Economía Keynesiana

Resumen

El objetivo de este trabajo es analizar los efectos sobre el bienestar que puede tener la política monetaria que establece la tasa de interés de referencia por intervalos discretos de tiempo. La hipótesis es que debido a que hay incertidumbre sobre diversas perturbaciones que tendrán lugar en el período en que se establece la tasa de interés referencial, esto puede causar una pérdida de bienestar social.
Para analizar el problema, se plantea un modelo donde el banco central minimiza una función de pérdida. Cuando hay perfecta certidumbre, se alcanza una frontera eficiente entre las varianzas de la inflación y el producto. Con incertidumbre el resultado es ineficiente. Esto implica la necesidad de discutir si sería conveniente que la tasa de interés se fijara de manera contingente.
La principal limitación del trabajo es tal vez que el modelo utilizado hace una gran cantidad de abstracciones, lo que le permite ser funcional, pero puede dejar de lado aspectos importantes de la realidad. Por otra parte, parece haber muy pocos trabajos, o tal vez ninguno, que traten el problema particular analizado este artículo.

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Resumen

Clasificación JEL: E31, E32, E43, E52
Palabras clave: inflación, variación de la inflación, variación de la producción, fijación de la tasa de interés, meta de inflación

1. Introduction

Currently, a large number of countries have established monetary policy inflation targets. In this context, the central bank sets a goal for inflation and establishes a rule of interest rates compatible with that objective.

A characteristic observed in most of the countries that have adopted this type of monetary policy, is that the nominal interest rate that arises from the described interest rate rule is established by a defined interval of time. In the United States, for example, eight meetings a year are scheduled for the Governing Board of the Federal Reserve Bank where it is possible to modify the federal funds rate. A very similar situation occurs in Mexico, where eight to nine annual meetings of the Governing Board of the Bank of Mexico are scheduled to analyze whether or not the interbank interest rate is modified, the main instrument of monetary policy in this country.

If in the interval between two meetings of the corresponding committee there are supply and/or demand shocks, the policy interest rate will remain unchanged. However, it is possible that an optimal monetary policy would have been to modify the aforementioned interest rate. It is also possible that once the corresponding committee meets again it is no longer necessary to make changes to the monetary policy. In any case, failure to act when necessary may have negative effects on well-being.

The objective of this paper is to analyze the effects on welfare that monetary policy that establishes the reference interest rate at discrete intervals of time may have. The hypothesis of the article is that because there is uncertainty about the supply and demand disturbances that will take place in the period in which the referential interest rate is established, this can cause a loss of social welfare.

The aforementioned inefficiency is manifested in the fact that the two factors that negatively affect a concept of well-being considered in the literature: the variance of inflation and the variance of the output, are equal to or greater than the value that would be observed if the interest rate could be modified at the moment in which supply and demand shocks occur, or if these disturbances could be predicted in advance.

The work is divided into three sections:

The first makes a brief review of the literature on what in the context of the new Keynesian economy literature is understood by optimal monetary policy, which assumes that the economy is on a frontier that minimizes the joint variances of inflation and output.

The second section establishes a model where one of two conditions occurs, or both: the monetary authority can foresee the supply and demand shocks that will occur between two consecutive meetings of the corresponding monetary policy committee, and/or these meetings are not scheduled and they carry out whenever any of the disturbances mentioned occurs. In this case we demonstrate that, in the presence of rational expectations, the central bank manages to reach a negative slope that minimizes a combination between the variance of inflation and the variance of output.

The third section modifies the assumption of the model in the previous section that the monetary authority can foresee the supply and demand shocks that occur between two of the programmed meetings. In this case it is shown that the central bank will not be able to be on the efficient frontier between the variance of inflation and the variance of the output, which implies, at least in theory, a loss of social welfare

Given the problem described, a possible solution discussed in the conclusions of the work is not to schedule meetings of the monetary policy committee to analyze and/or modify the interest rate of monetary policy, but to have these meetings in a completely contingent manner.

2. Brief Review of the Literature on the Monetary Policy of Inflation Targeting and the Relationship Between the Variance of Inflation and the Variance of Output

In 1958, A.W Phillips found a negative relationship between the growth of nominal wages and the unemployment rate in the United Kingdom. A little later, Samuelson and Solow (1960) proposed the existence of a tradeoff between inflation and unemployment, by which the monetary authority could choose to have more than one evil, inflation, generating a good, a fall of unemployment.

The tradeoff between inflation and unemployment was criticized a few years later by Phelps (1967) and Friedman (1968) as a relationship that does not exist in the long term. From these works arose the concept of natural rate of unemployment, a variable that does not change although inflation changes substantially.

For many years, the concept of the natural rate of unemployment permeated and became the mainstream in macroeconomics. There were some disagreements about the concept in works such as those by Tobin (1972), Akerlof, Dickens and Perry (2000) and Graham and Snower (2002), but, although very interesting, these articles had little impact on modifying what most economists of the mainstream thought.

After the international financial crisis of 2008-2009, some renowned economists have again questioned the concept of natural rate of unemployment, so it seems that soon there will be a great discussion on this issue (see Blanchard (2018)).

In 1979, John Taylor argued that although there was no long-term tradeoff between inflation and unemployment, there was another type of tradeoff, which occurred between the variance of inflation and the variance of output. Policies aimed at reducing the variance of inflation increased the variance of output, and vice versa.

In the early nineties, New Zealand and the United Kingdom adopted a new type of monetary policy which is known as inflation targeting.[2] Later, many other countries did the same. This type of policy assumes that the central bank will do everything possible to achieve the proposed target. However, in practice the central bank also aims to stabilize output along its trend.

Among the works that evaluate the effects of the inflation targeting policy are those of Bernanke and Woodford (2005), Mishkin and Schmidt-Hebbel (2007) and Blejer, et al (2000). These authors find that this type of policy helps to get lower inflation in the long term, have a lower response of inflation to oil and exchange rate shocks, strengthen the independence of monetary policy and obtain inflation levels close to the target. When disinflation has been achieved, the target of a long-term inflation that is no longer modified are more easily reached.[3]

With the advent of inflation targeting, Taylor (1994) took up his idea of 1979, noting that it is in the context of this type of monetary policy where there is a tradeoff between the variance of inflation and the variance of output, which occurs in an efficient frontier between these two variables (see also the work of Svensson (2008) for a similar conclusion).

The works of Ball (1997), Svensson (1997) and Fuhrer (1997) proposed models that had two assumptions: inflation targeting and a function of social loss that was composed of a weighted sum between the quadratic deviations of inflation with respect to its target

[2]New Zealand in 1990 and the United Kingdom in 1992.
[3]This is due to the fact that in the transition of the inflation targeting strategy the goals fall gradually.

and output with respect to what is called the natural output[4] By minimizing this loss function, the central bank derives a rule of interest rates, very similar to the one also proposed by Taylor (1993), which leads the economy to the efficient frontier between the variance of inflation and the variance of output.

Rotemberg and Woodford (1998) question whether the function of social loss assumed by the authors mentioned in the previous paragraph can be supported in microeconomic terms, and find that under certain conditions it can be sustained (see also Clarida, Gali and Gertler (1999) for a similar argument).

One conclusion of all these works is that the monetary policy of inflation targeting results in a necessary, but not sufficient, condition for monetary policy to be efficient. In this case, efficiency is defined as reaching the minimum frontier with a negative slope between the variance of the output and the variance of inflation.

Some authors differ from the idea expressed in the previous paragraph. In particular Cecchetti and Ehrman (1999) and Cecchetti et al (2002) document cases in which the inflation targeting policy led to a strong volatility of output, which they consider not necessarily optimal, since it is possible that the preferences of the central bank do not reflect the preferences of the general public (an issue that has also been dealt by Svensson (1997)).

Between the end of the 1990s and the first decade of the 21st century, a large amount of work was carried out to analyze the empirical relationship between the variance of inflation and the variance of output, which Taylor (1979) considered should be negative. Taylor himself (1994) points out that it is not always true that there is a negative relationship between these variances, which can occur when, even with inflation objectives, the central bank has a rule of interest rates that is not optimal, or that in terms of the subsequent works already analyzed, it does not minimize the social loss function.

Several authors find that at the empirical level there is a negative relationship between the variance of inflation and that of output: Bean (1998), for example, estimates an inverse relationship between these variables when the variance of inflation is high, but this variance reaches a point where it practically becomes a constant. On the other hand, Erceg, Henderson and Levin (1998) propose a model in which the negative relationship suggested by Taylor (1979) (1994) arises. This model is calibrated for the United States, finding that the tradeoff between the variances described is greater the more inflexible wages are.

Other works in this line, such as those of Apergis (2003) for Greece and Adolfson et al (2014) for Sweden, find that the adoption of a monetary policy with inflation goals led to a reduction in the variances of inflation and output. Therefore, an inefficient monetary policy was replaced by one that approached the efficient frontier proposed by Taylor (1979) (1994). In general, empirical studies that find a negative relationship between the variance of inflation and the variance of output imply in advance this relationship exists when based on a theoretical model. What these works really do is to find indirectly the parameters that shape the negative relationship described in a calibration exercise. In works that use econometric methods, the evidence on the negative relationship of the analyzed variances is much less convincing.

Following on from the above, the work of Lee (2002), which uses the GARCH method, does not find a definite relationship between the variances described for the years 1960 to 1979. From 1980 until 1998 it finds a negative but very flat relation, with strong increases in the variance of output that imply very small reductions in the variance of inflation, a result very similar to that found by Bean (1998) for the United Kingdom.

Using methods similar to those of Lee (2002), Mendoza (2003) does not find a definite

[4]Adolfson et al (2014) point out that the natural output would be the one that emerges from the trend of smoothing the GDP or its logarithm over time, while the potential output would be that which would be reached if prices and wages were perfectly flexible.

relationship between the variance of inflation and the variance of output in Mexico and in Turkey.

In a much-discussed paper, Ball and Sheridan (2005) analyze the success of the inflation targeting policy in the OECD countries. The authors conclude that the adoption of the aforementioned policy did not significantly reduce the variances analyzed in this paper. By reproducing the methodology of Ball and Sheridan (2005) for developing countries, Goncalves and Salles (2008) find that the countries that adopted the policy of inflation targeting did achieve a considerable fall in the volatilities of inflation and output relative to the countries that did not. Batini and Laxton (2007) come to a similar conclusion when analyzing another group of developing countries.

Other authors, such as Karanasos and Kim (2005) and Conrad and Karanasos (2008), who use variants of the GARCH technique, find more complex relationships between the variances described. In particular, Karanasos and Kim (2005) find that in Germany, the United States and Japan the relationship between the analyzed variances is positive. For their part, Conrad and Karanasos (2008) find that the variance of inflation positively affects the variance of output, while the latter negatively affects the variance of inflation.

The literature on inflation targeting and the relationship between the variance of inflation and the variance of output was very active from the mid-nineties to 2008. From then on, it was significantly reduced, which is probably due to the international financial crisis, which began that year, required the analysis of many experts who were dedicated to the topics of interest that this article discusses.

Notwithstanding, the inflation targeting policy remains in force in many countries and still presents challenges to address, such as the one we believe this work suggests.

3. The“Classic” Model of the New Keynesian Economy: There is a Tradeoff Between the Variance of the Inflation and the Variance of Output

One of the best known versions of the basic model of the new Keynesian economy assumes that prices are set in staggered form (Taylor (1980), Calvo (1983)), which generates the so-called new Phillips curve. Likewise, it is assumed that there is an IS function that negatively relates output to the interest rate and the central bank minimizes a social loss function, [5] which is a weighted sum of the quadratic deviations of inflation from its target and output of its natural or potential level.

The new simplest Phillips curve is the one proposed by Mankiw and Reis (2001), which is based on the work of Calvo (1983) and can be described as:

$$\pi_t = E_t\pi_{t+1} + \delta\left(y_t - y^*\right) + e_t \tag{1}$$

Where π_t is the current inflation, $E_t\pi_{t+1}$ is the expectation of inflation in the next period conditioned by the relevant information in the current period. The variable yt is the current output and y* is the natural or potential product, so $y_t - y^*$ is the output gap. The term et is a stochastic shock that is normally distributed and has zero mean. The variance is constant and takes the value σ^2.

The IS function of the economy is defined as:

$$y_t = H - br_t + v_t \tag{2}$$

This function establishes a negative consistent relationship between output (y) and the interest rate r. The parameter v is also a demand shock, which is normally distributed with zero mean and constant variance σ_v^2.

[5] Ball (1997), Fuhrer (1997) y Svenson (1997) trabajan con una curva de Phillps antigua y también con una función IS con la forma sugerida por Hicks (1937). Clarida, Gali y Gertler (1999) trabajan tanto con una nueva curva de Phillips como con una nueva función IS.

The parameter H could represent a term related to fiscal variables, where increases in government spending increase H and increases in the income tax rate reduce it. Other variables of the private sector, such as autonomous consumption, can also be included in the H parameter.

In the "classic" approach of the new Keynesian economy, the central bank has a well-defined objective that consists of minimizing a loss function (see for example Svensson (1997)), which is posed as:

$$L_t = \varphi(\pi_t - \pi^*)^2 + (1-\varphi)(y_t - y^*)^2 \tag{3}$$

Where π^* is the target inflation. $(\pi_t - \pi^*)$ is the inflation gap.

The loss function is quadratic, as in the Barro and Gordon model and (Barro and Gordon (1983)).[6] The best result for the central bank, and ideally for society, is to have a current inflation π_t equal to the goal π^* and output y_t equal to the natural one and y^*. Any other solution generates a positive loss.

The parameter φ represents the relative importance that the central bank assigns to stabilize inflation around its goal in comparison with the importance it gives to stabilize output $(0 < \varphi < 1)$. When φ is 1, the central bank is fully committed to inflation reaching its target. On the contrary, when $\varphi = 0$ $(1-\varphi = 1)$ the central bank only aims to stabilize output. In practice, the central institute takes into account both objectives.[7]

The loss function of the central bank in equation (3) has been discussed in the literature. Svensson (1997) says that ideally this function should represent society's preferences about output and inflation's stability and not the preferences of the monetary authority. If the central bank effectively takes into account the preferences of the society, then in carrying out its monetary policy will be reducing as much as possible the social loss.

However, Svensson (1997) also states that not always the social's loss function reflects the preferences of society, but it can only reflect the preferences of the members of the committee that decides the monetary policy. This problem has also been discussed by Cecchetti and Ehrman (1999) and by Cecchetti et al (2002), who argue that when the central bank sets a parameter φ in 1, it generates, as we will see soon, a volatility of output that hardly reflects what society really wants.

In the literature it has been discussed whether the loss function proposed by Ball (1997), Fuhrer (1997) and Svensson (1997) can really reflect welfare terms of private agents. According to Rotemberg and Woodford (1998), under certain assumptions, the loss function can reflect the welfare parameters of the private sector.

On the other hand, the inflation goal π^* and the natural output y^* are concepts that require an additional explanation:

In theory, the target of zero inflation would be a desirable long-term goal, since that implies price stability. However, most of the countries that have adopted the monetary policy of inflation targeting have inflation goals of between 0 and 3 %, with an average perhaps centered on 2 %. The reason why the inflation target is not exactly zero is related, according to a report by the Federal Reserve Bank of the United States,[8] with the fact

[6] An important difference of the approach proposed here with that of Barro and Gordon (1983) is that for these authors the goal of the product is greater than the natural product

[7] Svensson (2008) argues that while some central banks have as sole commitment to achieve low inflation, in practice they all worry about stabilizing output.

[8] The report of the Federal Reserve Bank says: "The Federal Open Market Committee (FOMC) judges that inflation at the rate of 2 percent (as measured by the annual change in the price index for personal consumption expenditures, or PCE) is most consistent over the longer run with the Federal Reserve's mandate for price stability and maximum employment. Over time, a higher inflation rate would reduce the public's ability to make accurate longer-term economic and financial decisions. On the other hand, a lower inflation rate would be associated with an elevated probability of falling into deflation, which means prices and perhaps wages, on average, are falling–a phenomenon associated with very weak economic conditions. Having at least a small level of inflation makes it less likely that the economy will experience harmful

Revista Mexicana de Economía y Finanzas Nueva Época, Vol. 14 Primer Número Especial Aniversario, pp. 541-557
DOI: https://doi.org/10.21919/remef.v14i0.422

that a lower goal of 2 % presents risks that the economy will fall in deflation, which could generate very harmful effects on economic activity. Another reason why in some countries the inflation target is not zero is because they are in the process of consolidating low inflation after having periods of high price growth two or three decades ago. This could be the case in Mexico, where the inflation target is 3 % (see Mishkin and Shmidt-Hebbel (2007) for a similar argument).

It is also very important to define the natural output. In this article we define this concept as the trend of GDP over time. Statistical techniques such as the Hodrick-Prescott filter, or other methodologies, allow obtaining this concept empirically. Adolfson et al (2014) point out that an alternative definition is that of the potential output, which would manifest itself when there are totally flexible prices and salaries.

In an analysis for Sweden, the authors mentioned in the previous paragraph affirm that depending on which definition of natural output is used (the trend or what would happen when there are totally flexible prices and salaries) the short-term substitution coefficient of the Phillips curve between inflation and the output gap (the parameter δ in the Phillips curve in equation (1) of this paper) changes dramatically. By the same token, it would also change the long-term tradeoff between the variance of inflation and the variance of output the long term, as we will see in the following pages.

Considering again the equations of the model already described, the primal problem of the central bank is to minimize the loss function (3) subject to equation (1) of the Phillips curve. This will generate a relationship between the output gap and the inflation gap $(\pi t - \pi *)$ or a concept similar to it. To obtain this function, the central bank has to establish a monetary rule for the interest rate, which is the well-known Taylor rule (Taylor (1993)).

The minimization of the social loss equation (3) subject to the Phillips curve equation (1) results in:

$$y_t - y^* = -\frac{\varphi\delta(E_t\pi_{t+1} - \pi^*)}{(\varphi\delta^2 + (1-\varphi))} - \frac{\varphi\delta e_t}{(\varphi\delta^2 + (1-\varphi))} \tag{4}$$

This equation, which we will call aggregate demand, shows a negative relationship between the output gap $(y_t - y*)$ and a similar term to the inflation gap $(\pi_t - \pi*)$, in this case $(E_t\pi_{t+1} - \pi*)$.

For equation (4) to be really the aggregate demand, the central bank has to establish the Taylor rule of interest rates, which arises from substituting equation (4) in the equation IS (2), which gives as a result

$$r_t = \frac{H - y^*}{b} + \frac{\varphi\delta(E_t\pi_{t+1} - \pi^*)}{(\varphi\delta^2 + (1-\varphi))b} + \frac{\varphi\delta e_t}{(\varphi\delta^2 + (1-\varphi))b} + \frac{v_t}{b} \tag{5}$$

Equation (5) is the Taylor rule of interest rates (the MP equation of the Romer and Taylor model, see Romer (2000), Taylor (2000)). When inflation and its expectations are equal and the stochastic shocks of supply and demand are zero:

$$r_t = \frac{H - y^*}{b} = r^* \tag{6}$$

Where r^* is the natural rate of interest, a term that was defined more than 100 years ago by the Swedish economist Knut Wicksell (Wicksell (1898)).

In the traditional analysis of the IS-LM model (Hicks (1937)), the natural interest rate depends on the H parameter, which is partly determined by fiscal policy. Higher

deflation if economic conditions weaken. The FOMC implements monetary policy to help maintain an inflation rate of 2 percent over the medium term.” Esta aseveración se encuentra en la página de internet `https://www.federalreserve.gov/faqs/economy_14400.htm`.

government spending, or a reduction in the income tax rate, are factors that increase the natural rate of interest and, through the Taylor rule (5), the present interest rate.[9]

The dual problem is to replace the Taylor rule (5) in the IS (2) function. The result is aggregate demand (4). Clearly, the Phillips curve (1) and the aggregate demand (4) are the relevant equations to solve for inflation (π_t) and output (yt), while the IS function solves for the interest rate rt.[10]

The way to solve for inflation and output is to replace the output gap (yt-y*) of aggregate demand (4) in the Phillips curve (1), which results in

$$\pi_t = \frac{(1-\varphi)}{\varphi\delta^2 + (1-\varphi)} E_t\pi_{t+1} + \frac{\varphi\delta^2}{\varphi\delta^2 + (1-\varphi)}\pi^* + \frac{(1-\varphi)}{\varphi\delta^2 + (1-\varphi)} e_t \tag{7}$$

This semi-reduced form indicates that inflation depends on future inflation expectations, the inflation target and the supply shocks et. In the absence of stochastic supply shocks, inflation would be a weighted average of its expectation in the future and of the central bank's target. For its part, the semi-reduced form of output is the aggregate demand itself (4), which does not depend directly on the level of inflation.

The reason why we call semi-reduced forms to equations (4) and (7) is because in models of rational expectations such expectations are endogenous. If they were exogenous, then the aforementioned equations would be reduced forms for inflation and output, but they are not properly so because the value of expectations must be resolved according to exogenous variables of the model.

To endogenize expectations, it is necessary to solve, first, equation (7) as an equation in forward differences, which can be carried out using a forward operator or a recursive substitution into the future (see Goldberg (1986) and the chapter 8 by McCallum (1989)).

To simplify the notation, we define:

$$j = \frac{\varphi\delta^2}{(\varphi\delta^2 + (1-\varphi))} \tag{8}$$

$$1 - j = \frac{(1-\varphi)}{(\varphi\delta^2 + (1-\varphi))} \tag{9}$$

Therefore, equation (7) becomes

$$\pi_t = (1-j)\, E_t\pi_{t+1} + j\pi^* + (1-j)e_t \tag{10}$$

If a recursive solution is made to the future, it must be considered, taking into account equation (10), that

$$E_t\pi_{t+1} = (1-j)\, E_t\pi_{t+2} + j\pi^* + (1-j)E_t e_{t+1} \tag{11}$$

Substituting (11) in (10), and repeating the exercise recursively for later periods, up to a period t + n:

$$\pi_t = (1-j)^n E_t\pi_{t+n} + j\pi^* \sum_{i=0}^{n} (1-j)^i + \sum_{i=0}^{n} (1-j)^{i+1} E_t e_{t+i} \tag{12}$$

[9] A very different result arises when the IS comes from optimization in consumption (see McCallum and Nelson (1999) or King (2000)). In this case, the natural rate of interest is equal to the subjective discount rate of the intertemporal utility function of consumers.

[10] It is interesting to note that the form of the IS function is irrelevant in the solution of inflation and output. A new IS function could be used, such as those used by Clarida, Gali and Gertler (1999), McCallum and Nelson (1999), King (2000) and Blanchard (2008) and the results for inflation and output and its variances would be exactly the same.

Taking the limit of this equation when n tends to infinity, we will find

$$lim_{n\to\infty}\pi_t = \pi^* + (1-j)e_t \tag{13}$$

This is because

$$lim_{n\to\infty}(1-j)^n E_t\pi_{t+n} = 0 \tag{14}$$

Transversality condition

$$j\pi^*\sum_{i=0}^{\infty}(1-j)^i = \frac{j\pi^*}{(1-(1-j))} = \pi^* \tag{15}$$

$$\sum_{i=0}^{\infty}(1-j)^{i+1}E_te_{t+i} = (1-j)e_t \tag{16}$$

The transversality condition (14) assumes that the expectation of inflation in a very distant future period ($E_t\pi_{t+n}$) has a finite value, so multiplying it by the factor $(1-j)^n$ takes a value very close to zero (this is because j is a parameter less than unity, which can be observed in equations (8) and (9)).

For its part, and again taking into account that j is a parameter less than unity, the value $\sum_{i=0}^{\infty}(1-j)^i$ in equation (15) converges to factor $1/j$, which multiplied by $j\pi^*$ yields π^*.

Finally, the sum $\sum_{i=0}^{\infty}(1-j)^{i+1}E_te_{t+i}$ is simply equal to $(1-j)e_t$, since $E_te_t = e_t$ and $E_t(e_{t+i})$ for any period i positive is zero. This is because it is assumed that supply shocks have zero mean and they are not autocorrelated.

The difference equation (10) is a stable equation, because the parameter j is less than unity. It is equivalent to a first-order difference equation that is solved in the traditional way when looking at the past.

A possible alternative solution to equation (10) is to use a forward operator, whose form has been shown to arrive at a known solution (see Goldberg (1986)). In this case, it is defined:

$$E_t\pi_{t+1} = L^{-1}\pi_t \tag{17}$$

Where L-1 is the forward operator

Substituting (17) in (10) we get:

$$\pi_t = \frac{j\pi^* + (1-j)e_t}{(1-(1-j)\,L^{-1})} \tag{18}$$

In the literature on the solution of difference equations (Goldberg (1986)) it is known that

$$\frac{x_t}{(1-(1-j)\,L^{-1})} = \sum_{i=0}^{\infty}(1-j)^i E_t x_{t+i} \tag{19}$$

Where x is any variable, it can even be a constant.

Therefore, when applying (19) to (18) we get:

$$\pi_t = j\pi^*\sum_{i=0}^{\infty}(1-j)^i + (1-j)\sum_{i=0}^{\infty}(1-j)^i E_te_{t+i} = \pi^* + (1-j)e_t \tag{20}$$

That is the same solution for inflation that the one shown in (13).

For the totally reduced solution for output, the aggregate demand (4) is rewritten as:

$$y_t - y^* = -\frac{j}{\delta}\left(E_t\pi_{t+1} - \pi^*\right) - \frac{j}{\delta}e_t \tag{21}$$

But using equations (13) o (20) it is possible to show that

$$E_t \pi_{t+1} = \pi^* \tag{22}$$

Therefore, the reduced form for output is:

$$y_t = y^* - \frac{j}{\delta} e_t \tag{23}$$

Positive supply shocks ($e_t < 0$) generate a situation where output increases above its natural level and inflation falls. However, given the assumptions of the model, the impact of these shocks is diluted in one period. It should be noted that demand shocks do not affect the system of output and inflation because, when they occur, the Taylor rule indicates that the interest rate must be increased to cancel its effect on inflation and output (see equation (5))).

We will now show that there is a tradeoff between the variance of output and the variance of inflation. Output will vary the more the central bank is committed to keeping inflation close to the target π^*, and vice versa. The non-conditional expectation for inflation in equations (13) or (20) is π^*, since the non-conditional expectation for the stochastic supply shock is zero. Therefore, the variance of inflation is:

$$Var\,(\pi_t) = E(\pi_t - E\,(\pi_t))^2 = E(\pi_t - \pi^*)^2 = (1-j)^2 \sigma^2 \tag{24}$$

At the same time, the non-conditional expectation of the product in equation (23) is y^* for the same reason that was explained for the previous equation, then:

$$Var\,(y_t) = E(y_t - E\,(y_t))^2 = E(y_t - y^*)^2 = \frac{j^2}{\delta^2} \sigma^2 \tag{25}$$

The parameter j is directly related to φ, which represents how much commitment the central bank has with the inflation target compared with the objective of stabilizing output. When φ=1, then j=1 and the central bank is solely committed to keeping inflation at its target π^*. When $\varphi = 0$, j=0 and the central bank only wants to keep output at its natural level y*.

There is a monotonic relationship between j and φ, which can be observed by taking the derivative of j with respect to φ in (8):

$$\frac{dj}{d\varphi} = \frac{\delta^2}{(\varphi\delta^2 + (1-\varphi))^2} > 0 \tag{26}$$

$$\frac{dVar\ (\pi_t)}{dj} = -2(1-j)\sigma^2 < 0 \tag{27}$$

$$\frac{dVar(y_t)}{dj} = 2\frac{j}{\delta^2}\sigma^2 > 0 \tag{28}$$

The more the central bank commits itself to the inflation target π^* (the larger are φ and j), the lower the variance of inflation and the greater the variance of output.

The highest possible variance of output and the minimum variance of inflation occurs when $\varphi = 1 = j$. In this case, the central bank is fully committed to the inflation target, then

$$Var\,(\pi_t) = 0 \tag{29}$$

$$Var\,(y_t) = \frac{\sigma^2}{\delta^2} \tag{30}$$

The highest possible variance of inflation and the minimum possible variance for ouput occurs when $\varphi = 0 = j$, this happens when the central bank wants to keep output at all costs at its natural level y*.

$$Var\left(\pi_t\right) = \sigma^2 \tag{31}$$

$$Var\left(y_t\right) = 0 \tag{32}$$

The function that shows the tradeoff between the variance of inflation and the variance of output surges solving for j in (25) and substituting in (24), which results in:

$$Var\left(\pi_t\right) = \sigma^2 - 2\left(Var\left(y_t\right)\right)^{1/2}\delta\sigma + Var(y_t)\delta^2 \tag{33}$$

It is possible to prove that this function has a negative slope, since the first derivative of the variance of inflation with respect to the variance of output is:

$$\frac{d(Var\left(\pi_t\right))}{d(Var\left(y_t\right))} = -\left(Var\left(y_t\right)\right)^{-\frac{1}{2}}\delta\sigma + \delta^2 \tag{34}$$

For this derivative to be negative, it is necessary that

$$-\left(Var\left(y_t\right)\right)^{-\frac{1}{2}}\delta\sigma + \delta^2 < 0 \tag{35}$$

But that implies

$$\frac{\sigma^2}{\delta^2} > Var(y_t) \tag{36}$$

Which is true because the maximum variance of output is exactly σ^2/δ^2.

On the other hand, the second derivative is positive, which shows that there is a convex relationship in the plane where the variance of inflation is on the vertical axis and the variance of output is on the horizontal axis. The latter can be observed by taking the second derivative in (34):

$$\frac{d^2 Var(\pi_t)}{d(Var\left(y_t\right))} = \frac{1}{2}(Var\left(y_t\right))^{-3/2}\delta\sigma > 0 \tag{37}$$

Equations (34), (36) and (37) show that in the plane where the variance of inflation is on the vertical axis, and the variance of output is on the horizontal axis, the relationship between these two variables is inverse and convex to the origin.

On the other hand, the expected value of the loss function (3) is a linear relationship between the variance of inflation and the variance of output. This can be observed by applying the non-conditional expectation operator to equation (3) and considering the results already analyzed, where the expected non-conditional values for inflation and output are, respectively, π^* and y*.

$$\begin{gathered} E\left(L_t\right) = \varphi E(\pi_t - \pi^*)^2 + (1-\varphi)\,E(y_t - y^*)^2 = \\ \varphi E(\pi_t - E\left(\pi_t\right))^2 + (1-\varphi)\,E(y_t - E\left(y_t\right))^2 \\ = \varphi Var\left(\pi_t\right) + (1-\varphi)\,Var(y_t) \end{gathered} \tag{38}$$

The endogenous solution for the variance of output and the variance of inflation occurs in the tangency between the function of variances (33) and the expected function of loss (38). There is only one variance function, but there is a dense map of expected loss functions, all of them linear.

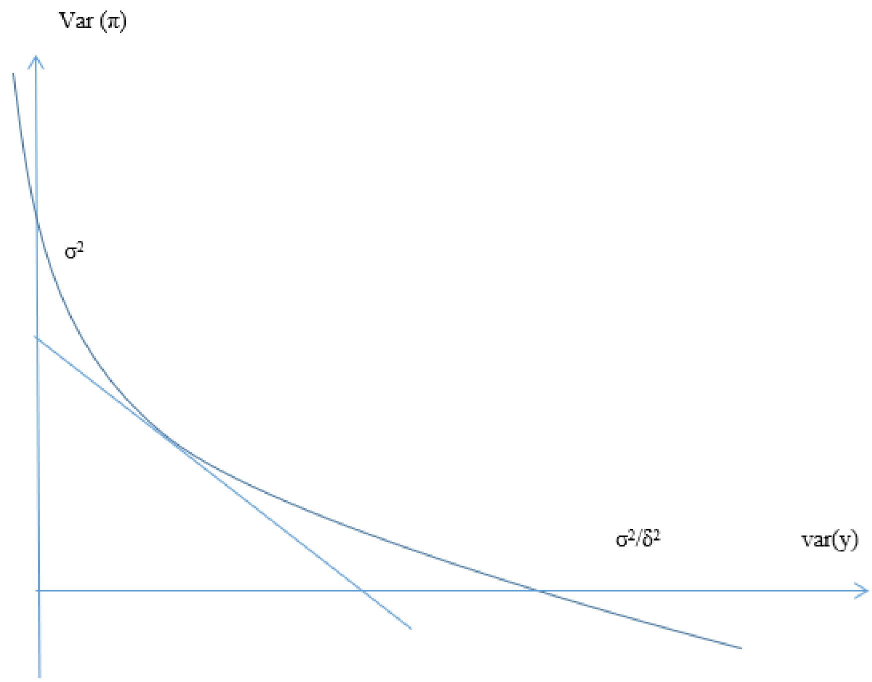

Figure 1. The tradeoff between the variance of inflation and the variance of output in the "classical" model of the new Keynesian economics.

3. An Alternative Approach in the Spirit of the New Keynesian Economy: In Presence of Imperfect Information the Negative Relationship Between the Variance of Inflation and the Variance of Output May Disappear

As we have seen, by minimizing the loss function the central bank has to establish a rule of interest rates, the famous Taylor rule (equation (5)) (Taylor (1993)). When the central bank anticipates all the possible supply and demand shocks that will occur during the period in which a level of interest rates is established, the interest rate rule includes these disturbances. We consider it convenient to repeat equation (5):

$$r_t = \frac{1}{b}(H - y^* + \frac{\varphi\delta}{(\varphi\delta^2 + (1-\varphi))}(E_t\pi_{t+1} - \pi^*) + \frac{\varphi\delta}{(\varphi\delta^2 + (1-\varphi))}e_t + v_t) \tag{39}$$

In real life, information is limited. Central banks set levels of interest rates at discrete points of time, usually at meetings of the governing board of said institutions. Assuming that the periods last symmetrical intervals, for example one month and a half or three months, it is clear that if the central bank sets the interest rate at the beginning of the period, or at the end of the previous period, it will not be able to forecast at least some supply and demand shocks that will occur during the time the interest rate remains fixed.

In this section we assume the opposite case to that of the first section. The central bank sets the interest rate for period t in period t-1. The governing board of the central bank does not know at that time whether there will be supply or demand shocks in period t. The best forecast for these shocks is that they have a value of zero, because the assumptions of the previous section that is the conditional and non-conditional expectations of those disturbances.

Usually, central banks set nominal interest rates and not real rates. Given the information that the central bank has in the period t-1, a reasonable proxy of the optimal rule

(39) would be to establish the nominal interest rate as follows:

$$R_t = E_{t-1}\pi_{t+1} + \frac{1}{b}(H - y^* + \frac{\varphi\delta}{(\varphi\delta^2 + (1-\varphi))}\left(E_{t-1}\pi_{t+1} - \pi^*\right)) \tag{40}$$

Where R is the nominal interest rate that will prevail in t but that is established at the end of period t-1.[11]

However, the definition for the real interest rate is:

$$r_t = R_t - E_t\pi_{t+1} \tag{41}$$

The substitution of (40) in (41) in the IS (2) equation results in:

$$y_t = y^* - \frac{\varphi\delta}{(\varphi\delta^2 + (1-\varphi))}\left(E_{t-1}\pi_{t+1} - \pi^*\right) - b(E_{t-1}\pi_{t+1} - E_t\pi_{t+1}) + v_t \tag{42}$$

That is an aggregate demand slightly different from that found in equation (4) in the problem of minimization of the social loss function of the previous section.

Substituting this aggregate demand (42) in the new Phillips curve (1) we obtain:

$$\pi_t = E_t\pi_{t+1} - jE_{t-1}\pi_{t+1} + j\pi^* - b\delta\left(E_{t-1}\pi_{t+1} - E_t\pi_{t+1}\right) + \delta v_t + e_t \tag{43}$$

Where j is defined exactly the same as in equation (8) of the last section.

Applying the conditional expectation operator in t-1 to equation (43) and using the law of iterated expectations, where

$$\begin{aligned} E_{t-1}E_t\pi_{t+1} &= E_{t-1}\pi_{t+1} \\ E_{t-1}E_{t-1}\pi_{t+1} &= E_{t-1}\pi_{t+1} \end{aligned} \tag{44}$$

We get

$$E_{t-1}\pi_t = (1-j)\,E_{t-1}\pi_{t+1} + j\pi^* \tag{45}$$

It is possible to solve this difference equation as in the previous section, by a recursive substitution method or by using a forward operator. If we choose this last method, we find that

$$E_{t-1}\pi_t = \frac{j\pi^*}{\left(1 - (1-j)\,L^{-1}\right)} \tag{46}$$

But for the known solution that has the use of this forward operator (see Goldberg (1986)), it can be concluded that

$$E_{t-1}\pi_t = j\pi^* \sum_{i=0}^{\infty} (1-j)^i = \frac{j\pi^*}{(1-(1-j))} = \pi^* \tag{47}$$

This equation in first order differences of expectations has a stable solution because j <1.

Given the equation (45) it is also true that in the period t+1

$$E_t\pi_{t+1} = (1-j)\,E_t\pi_{t+2} + j\pi^* \tag{48}$$

Therefore, doing exactly the same procedure above:

$$E_t\pi_{t+1} = \pi^* \tag{49}$$

[11]Strictly for this model Rt is set in t-1. It is not possible to algebraically distinguish between the beginning and the end of a period in discrete time.

Applying to this solution (49) the operator of expected value in t-1 and taking into account the law of iterated expectations:

$$E_{t-1}E_t\pi_{t+1} = E_{t-1}\pi_{t+1} = \pi^* \tag{50}$$

Substituting (49) and (50) in (43) is the reduced form for inflation

$$\pi_t = \pi^* + \delta v_t + e_t \tag{51}$$

Also, replacing (49) and (50) in the aggregate demand (42), we find the reduced form for output.

$$y_t = y^* + v_t \tag{52}$$

In this case, positive demand shocks (v>0) increase inflation and output above their long-term values, while supply shocks only affect inflation.

The non-conditional expected values for output and inflation are y* and π^*, respectively, from which it follows that:

$$Var\,(\pi_t) = E(\pi_t - E\pi_t)^2 = E(\pi_t - \pi^*)^2 = \delta^2\sigma_v^2 + \sigma^2 \tag{53}$$

$$Var\,(y_t) = E(y_t - Ey_t)^2 = E(y_t - y^*)^2 = \sigma_v^2 \tag{54}$$

The variances of inflation and output are different from the case in which there is perfect information. When the Taylor rule for period t is set at t-1, the analyzed variances have no relation to central bank preferences. Therefore, there is not any tradeoff between them.

It is relatively easy to prove that the limited information of the central bank generates an inefficient solution with respect to the case described in the previous section. This can be observed by making a diagram where the variance of inflation is on the vertical axis and the variance of the product on the horizontal axis.

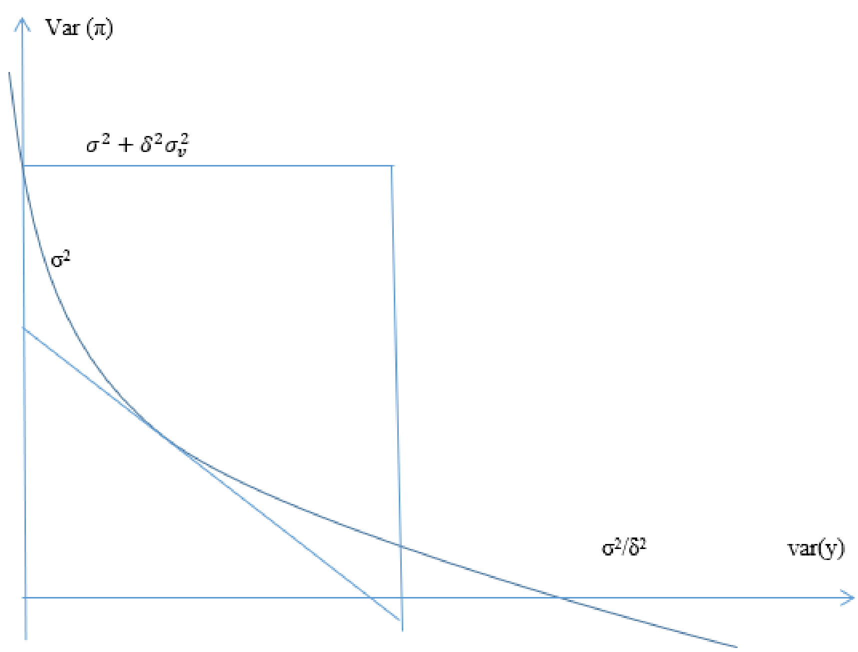

Figure 2. Solution for the variances of inflation and output when there is limited information and its comparison to the case of perfect information.

The solution for the case of imperfect information occurs at the point where the vertical and horizontal straight lines that are in the diagram intersect, while the solution for the case of perfect information occurs in the tangency of the convex efficient frontier and the line of negative slope that represents the expected loss function.

Geometrically, the equilibrium point of the variances in the case of imperfect information is always to the northeast of the solution with perfect information. We know this with certainty, because the variance of inflation in the case of imperfect information is $\delta^2\sigma_v^2 + \sigma^2$, which is always greater than the maximum variance that inflation can obtain when there is perfect information, which is σ^2. In the case of imperfect information, it could happen the variance of output could be smaller than the one that takes place in perfect information, but even so the equilibrium point of the variances would be above the efficient frontier.

4. Conclusions

The new Keynesian economy has proposed the existence of a tradeoff between the variance of inflation and that of output. Central banks that are strongly committed to meeting their inflation target will generate a greater variance of output.

In this paper we show that, when the central bank does not have perfect information, the inverse theoretical relationship between the variances of inflation and the one of output can be broken. This sometimes happens because central banks set interest rates for well-defined time intervals. If the establishment of the interest rate occurs before various supply and demand shocks occur, this information cannot be incorporated into the monetary policy instrument. In that case, the effect of these disturbances on the variances and inflation and the one or output can be very different from the case where the central bank can foresee these shocks.

The result is that under perfect information, the solution is sub optimal and, at least in theory, reduces social welfare in relation to the case where there is perfect information.

Would it be convenient for the central bank's board of governors to stop having periodic meetings in which the interest rate can be modified and act in a purely contingent manner? The results of this work support this proposal in some way. However, perhaps that would cause uncertainty in financial markets, so a better solution would be to set intervention thresholds. If supply and demand shocks are small, the loss in welfare of not being able to forecast them is also small. If they are large, it would be better to intervene contingently.

Even if the central bank could modify the interest rate at all times, the problem that we have analyzed would arise if there is not complete information about the state of the economy. Sometimes, it is not known what is happening with the economic activity and official figures about what is really happening at any given time may take months to be published. Hence, the use of coincident and advanced indicators is crucial for decision makers.

The model used in this article abstracts from many aspects of reality. This allows it to be functional, while at the same time it can rescue the main aspects of the relationship between the variances of inflation and output that have been analyzed in more complex contexts. Its relative ease and functionality also allow the model to explore relevant aspects in the presence of uncertainty, which is the main objective of the work. Among the great advantages it has is to offer exact analytical solutions to the endogenous variables, something that could not be achieved if there were, for example, non-linear elements.

However, it must be recognized that due to the abstraction that it does, the presented model is not designed to carry out simulations or calibrations with data from specific countries, which constitutes a limitation and, at the same time, a challenge to be overcome in the future.

An important point to note is that, as far as our possibilities have come, we have not been able to find academic works that analyze the problem presented here, which is the uncertainty generated by the way in which decision-making is designed to modify the rate of interest of the monetary policy.

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