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The Innovation-Absorption Dichotomy, Distribution, and Efficiency

David **Mayer-Foulkes***

Abstract

The fact that some firms innovate while others absorb technologies creates income differences and inefficiencies both within and between countries.

We model the industrial market economy, characterized by the coexistence of: 1) large firms with innovation, as in manufacturing and technology, concentrating income and wielding market power, with profits accruing to a small mass of people; and 2) small, approximately competitive firms with little innovation capacity, mainly absorbing technologies. This economy is unequal and inefficient. Free market policies are suboptimal in levels, growth rates, wages, capital accumulation and equity. Their levels depend on the steady state of the innovation-absorption dynamics between the two sectors, and can all be improved by taxing profits and subsidizing both innovation and absorption, or by lowering profit margins through a market power tax with zero equilibrium revenues. Innovators access a higher return to their investment than the interest rate.

Now, consider a second industrial economy whose large scale sector absorbs technologies from the first industrial economy. The first is a developed technological leader and the second an underdeveloped technological follower, that might grow in parallel or diverge. The underdeveloped industrial economy is also inefficient and unequal. Moreover, its small scale sector lags relatively further behind than in the developed economy.

Technological policies for equitable, pro-poor growth in industrial economies, developed or underdeveloped, must be two-pronged: supporting both innovation and absorption.

Palabras Clave: Crecimiento, Distribución, Eficiencia, Tecnología, Competencia, Poder de Mercado
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* Centro de Investigación y Docencia Económicas.

1. Introduction

In clarifying the functioning of modern developed and underdeveloped economies, particularly inequality, it is necessary to specifically address them as *industrial market economies*. The reason is that two types of economic sectors coexist in developed and underdeveloped industrial economies. First, sectors with intense innovation, such as the manufacturing and technology sectors. These consist mainly of large firms that concentrate income and are understood in terms of market power and models of endogenous technological change (see for instance the research line generated by Aghion and Howitt, 1992, 1998). In underdeveloped economies, these large firms absorb technologies from developed economies when they innovate. Second, sectors with a much smaller innovation capacity, which mainly consist of many small firms, and can be thought to be approximated by competitive general equilibrium models, with income more equally distributed. These later sectors improve their productivity by adopting technologies developed in the innovative sectors[1]. Distinguishing between innovation and absorption in industrial economies gives Nelson and Phelps' (1966) " absorptive capacity" its effective weight.

The purpose of this article is to extend the Mayer-Foulkes (2015a) (abbreviated MF) industrial market economy model to a two country model of industrial development and underdevelopment.[2] As in MF, each country's economy consists of a large and a small scale sector, the first innovative, with monopolistic competition, and the second absorptive, with perfect competition. I use a standard model of development and underdevelopment such as those developed by Howitt and Mayer-Foulkes (2005) (HMF) and Aghion, Howitt, and Mayer-Foulkes (2005) (AHM). In these models underdeveloped countries innovate by absorbing knowledge from developed technological leaders. Thus, here there are two countries. Country 1's large scale sector is the technological leader. When firms in Country 2 innovate, they incorporate these technologies for profit in a costly process that ranges from implementation to R&D, as in HMF and AHM. In turn, the small sector absorbs technologies from the large scale sector, for simplicity only from its own country.

The heart of the industrial market economy model is an industrial, technological, or mass production sector characterized by ongoing technological innovation. Innovation is motivated by the acquisition of market power and generates large firms spanning important portions of their markets. The controlling ownership of these large firms concentrates in relatively few hands that can appropriate most of the profits resulting from this market power, thus generating income concentration. Market power also generates both static and dynamic inefficiencies in innovation and production. Hence the heart of the industrial market economy simultaneously displays growing productive efficiency, wealth concentration, and economic inefficiency.

Now, for various reasons not every economic sector can finance innovation by obtaining sufficient profit margins over significant market sectors. An important

1. We refer interchangeably to absorption, technological diffusion, adoption, and implementation.

2. See also Mayer-Foulkes and Hafner, 2016. In that paper we use *mass market economy* instead of *industrial market economy*.

proportion of the working population in industrial market economies, developed or underdeveloped, is employed by many small firms that do not innovate significantly. Instead, these small-firm, non-innovating sectors, including self-employment and informal economic activity, improve their productivity by expending effort on absorbing the technologies developed by the industrial sector, which functions as its technological leader. The model presented here brings to bear insights of underdevelopment (HMF, AHM) and globalization (Mayer-Foulkes, 2014) to the inner working of both developed and underdeveloped industrial economies.

Relative to each other, the large and small scale sectors display opposite characteristics. While the first is innovative and displays market power, the second absorbs technologies and is competitive. Each may thus require differently motivated public policies in physical and human capital, technology, infrastructure and so on. The aggregate product of the industrial market economy is a function of the technological levels of both of the sectors. This aggregate income, net of profits, in turn determines wage levels, skilled and unskilled. In the industrial market economy, the wage level is thus a positive function of the technological levels of both the leading industrial sector and the lagging small firm sector, and a negative function of market power.

Including the technologically lagging small firm sector in the analysis allows wages to be understood in terms of the full dynamics of innovation and absorption that determine them. Conversely, the interaction of innovation profits in the large scale sector with the interest rate on capital in the small scale, competitive sector, provides a context for understanding the role of the stock market in bringing forward innovation profits, by capitalizing innovation income streams according to the prevailing interest rate. This can also be an important source of differentiated returns according to the scale of investment in the financial system, which plays a central role in Piketty's (2004) argument on inequality.

Neither a general competitive equilibrium model nor a standard model of endogenous technological change includes the full set of features mentioned above. Yet these features are necessary ingredients for understanding a series of urgent issues that are deeply affected by the contradictory impacts of innovation and competition on welfare and distribution, such as global income concentration, income inequality, informality, pro-poor growth, the increased political influence of large corporations under deregulation, sustainability in the face of both poverty and corporate power, the global economic business cycle, and so on.

Market institutions establishing property rights and enforcing contracts allow for the existence of private production under competition forms that can be either competitive or characterized by market power. Free market policies allowing prices to be set without government interference allow the free play of both competition and market power. MF show that free market policies are suboptimal for industrial market economies, in levels, growth rates, wages, capital accumulation and equity. Profits also undo Pareto optimality by making industrial products more expensive, therefore misallocating resources towards non-innovation-intensive goods, in both innovation and consumption. MF also describes public economic policies that can improve on free market policies by simultaneously promoting both equity and productivity. The first promotes both industrial and small scale technologies by supporting innovation and absorption using taxes on profits. The second set of policies promotes efficiency and distribution by taxing excess profit *rates*, therefore rewarding *production* rather than profit rates by discouraging the exercise of market power. The equilibrium taxation revenue in this case is zero. The third addresses the public good features of innovation in the small scale sector. This can be done by enhancing the role of education as a catalyst of technological absorption and by promoting the interaction between universities and the small scale sector to produce needed innovations and to reduce the repetition of innovation in this sector.

The model goes quite a long way in explaining the inequality pointed out by Piketty (2014) for industrial market economies. The reasons are the following. First, in the model Piketty's r in fact includes the profit rate, which is even more easily greater than the growth rate g. Second, the income concentration process described in the model works in terms of the returns to real rather than financial investments. It is not only that large accounts can get a preferential rate of return in financial investments. It is also that large real investments can access the profit rate through innovation rather than just the interest rate through capital investment. While discussing the historical aspects of Piketty's (2014) work is beyond the scope of this paper, I would hypothesize that convergence to equilibrium inequality levels or capital to income ratios is faster than posited by Piketty, and responds significantly in a couple rather than in quite a few decades to substantial changes in profit level determinants. Thus, while the two World Wars may have had the most salient negative impacts on capital accumulation, other changes such as the rise and fall of the economic framework of the Great Prosperity (including taxes on profits, human capital investment, financial regulation and welfare), or the epochal changes in globalization of the last thirty years, have also had highly significant impacts.

MF point out that the large scale sector gained ascendancy in the late 19th Century with the consolidation of the Second Industrial Revolution, has been the norm in 20th Century capitalism, and increased its power and impact on inequality under globalization. The industrial market economy model predicts not only static but also dynamic inefficiency in innovation and absorption. In the case of innovation, for the quality ladder case considered here, high markups give a lower weight to production costs, and therefore to the benefits of innovation. In the case of absorption, its operation in a competitive setting implies insufficient incentives for technological absorption. Therefore incentives arise for financing absorption as a public good. A subsidy to higher education is in itself an example.

One of the theoretical classics in the literature poses the existence of an inverted-U relationship in competition and innovation (Aghion et al, 2005), thus suggesting that there may be an optimal level of market power. MF define a market power tax that can provide an instrument for selecting some such designated level of market power.

In the next section I develop the model for the static industrial market economy. In the following section I introduce technological change in a two country model that simultaneously defines lags between and within countries. The analysis of taxes and subsidies for improving efficiency is included in each of the sections. Finally I discuss and conclude.

2. The model

I first define a two sector model of the industrial market economy that describes its main macroeconomic features and can be used to analyze a series of issues. These main features include a leading industrial sector generating productive efficiency and growth through technological innovation; market power in this sector; wealth concentration deriving from this sector; a complementary, technologically lagging competitive economic sector responsible for a considerable portion of production and employment; technological adoption in this sector; a wage level determined by the combined productivity of both sectors, net of profits.

2.1 The static economy

Consider an economy with two sectors *L* and *S* that produce a continuum of tradeable goods indexed by $\eta \in [0,1]$, where each η refers to a product. Large scale sector goods $\eta \in \tau_L = [0,\xi]$ use a mass production technology and are therefore modelled with all production concentrated on a single large producer that is able to make a profit, while small scale sector goods $\eta \in \tau_S = [\xi,1]$ are produced on the small scale, with constant returns to scale, therefore modelled with infinitely many small, identical, competitive producers. In each sector technological change is endogenous, with differences due to the different competition structures. For simplicity we abstract from innovation uncertainty and assume that innovation is symmetric within each sector *L* and *S*. Thus we are assuming goods $\eta \in \tau_j$ in each sector *j* have the same technological level A_{jt}, $j \in \{L,S\}$.

Innovation occurs as follows. In the large scale sector *L* there is for each good $\eta \in \tau_L$ a single, infinitely lived innovator who invests in innovation and becomes a national monopolist, producing in the presence of a competitive fringe (as in Howitt and Mayer-Foulkes, 2005). Innovation is cheaper for the producing incumbent than for the competitive fringe, and she therefore has an innovation advantage. Her monopoly therefore persists indefinitely. By contrast, in the small sector *S* anybody can innovate, so as to reap the productive benefits of new technologies, namely the availability of returns to production factors, in this model labor.

We assume that small producers can produce any good, while large producers can only produce goods in sector τ_L for which mass production technologies are available that are more productive than small scale technologies.

2.1.1 Consumption

Let the instantaneous consumer utility $U=U(C_t)$ depend on a subutility function C_t for an agent consuming $c_t(\eta)$ units of goods $\eta \in$ [01, according to the Cobb-Douglass function

$$\ln\left(C_t\right) = \int_0^1 \ln\left(c_t(\eta)\right) d\eta. \qquad (1)$$

As we shall see, given a constant budget and goods at a constant price, this utility function expresses Cobb-Douglass preferences for variety.

Suppose a consumer has a budget B_t for purchasing goods produced in the large and small scale sectors. We assume large and small scale sector goods $\eta\in\tau_L$, τ_S are symmetric so have common prices p_{Lt}, p_{St}. Since the composite good kernel (1) is Cobb Douglass, consumers dedicate the same budget to each good $\eta\in[0,1]$. The budget for each good is therefore also B_t, so the quantity bought of each type of good produced in the large scale sector is $c_t^L=\frac{B_t}{p_{Lt}}$, and in the small sector is $c_t^S=\frac{B_t}{p_{St}}$. Hence the subutility (1) of the goods produced is

$$\ln C_t=\xi\ln\frac{B_t}{p_{Lt}}+(1-\xi)\ln\frac{B_t}{p_{St}}, \qquad (2)$$

which implies

$$C_t=\frac{B_t}{p_{Lt}^{\xi}p_{St}^{1-\xi}}. \qquad (3)$$

Given a budget $p_{Lt}^{\xi}p_{St}^{1-\xi}$, the amount of composite good produced is C_t=1. Letting the composite good be the numeraire, this costs 1, so

$$p_{Lt}^{\xi}p_{St}^{1-\xi}=1. \qquad (4)$$

2.1.2 Two kinds of producers

To motivate the model, let us examine the two sectors of production, *L* and *S*, in more detail.

Applying technologies of mass production requires producing for a sizeable proportion of the market. Producers are therefore not small. The markets for goods produced using large scale technologies therefore exhibit market power and profits, as in monopolistic competition. At the same time these profits provide incentives for innovation, as a means to increase profits and maintain market power. In this sense, market power is an endogenous function of innovation. Rather than assuming monopoly, it is customary to assume the existence of a competitive fringe that can enter the market at a lower level of productivity but nevertheless limits the markup that the incumbent can obtain. Here we assume this competition lies in the large scale production sector. We limit ourselves to innovation as the source of market power, though fixed costs and increasing returns to scale are also present in mass production. In constructing the model we attempted to use these in addition to innovation, but both gave rise to mathematics that were too complex for the present purpose.[3]

By contrast, small-scale production occurs in small firms that we will assume are price takers. Nevertheless, these firms will also invest to improve their productivity. However, the returns to this investment will not be profits but labor productivity. The two processes of innovation will be thought to be qualitatively different, the first operating on a large scale and truly innovating, the second operating in a small scale and adopting technologies developed by the first.

The previous two paragraphs explain why, while the large and small-scale production sectors are quite different, their production functions can for simplicity be represented by similar functions. The two sectors are only distinguished by their competitive context.

Definition 1. *The production function for goods* $\eta\in\tau_j$ *in sector* $j\in\{L,S\}$ *is:*

$$y_{jt}(\eta)=A_{jt}l_{jt}(\eta), \quad j\in\{L,S\}. \qquad (5)$$

Here $y_{jt}(\eta)$ represents the quantity produced of good $\eta\in\tau_j$. A_{jt} is the technological level in each sector. $l_{jt}(\eta)$ is the quantity of labor input.

2.1.3 Choice of production quantities

We assume for simplicity that the only input in production is a single kind of homogeneous labor, and that the labor market is perfect. Therefore there is a single wage level w_t across sectors in any given country.[4]

3. It is worth noting that in the case of fixed costs two equilibria arise for the two sector economy developed here, as in Murphy, Shleifer, and Vishny (1989). Also, increasing returns to scale are related to the ratio of employment demand between the two sectors and therefore to wage levels.

4. We could introduce heterogenous abilities, with each worker's labor measured in effective labor units. The model would remain essentially unchanged, with workers now earning in proportion to their ability, and possibly one sector selecting higher abilities first. We could similarly also introduce human capital, with acquired abilities measured in effective labor units. In a perfect human capital market, net lifelong earnings would be constant, and again the model would remain essentially unchanged. We assume the labor market is homogeneous and perfect because our purpose is to observe how the economy-wide wage level depends on the combination of technological levels in the large and small scale sectors, independently of any market imperfections. It therefore depends on the dynamics of innovation and absorption. It would even be possible for ability or human capital to be useful only in one sector, for example the large scale sector. If again this sector selected for high ability or human capital first, these workers would be employed in the large scale sector and earn more, in proportion to their ability or human capital, while workers in the small scale sector would earn the wage for a single unit of labor. The model would still remain essentially unchanged, in that the equilibrium wage would continue to be an analogous function of both technological levels.

In the case of small producers one unit of good $\eta \in \tau_S$ is produced competitively by infinitely many firms. The wage is equal to the income from selling the product of one unit of labor, $w_t = p_{St} A_{St}$, so the price can be written

$$p_{St} = \frac{w_t}{A_{St}}. \quad (6)$$

Suppose the constant expenditure across goods is z_t. In each sector $\eta \in \tau_S$ let $l_{St}(\eta)$ be the aggregate employment of all of the firms producing this good. Since the number of units produced is $c^S_{jt} = \frac{z_t}{p_{St}} = A_{St} l_{St}(\eta)$, the labor quantity is constant in η, so we drop η from the notation, and

$$l_{St} = \frac{z_t}{p_{St} A_{St}}. \quad (7)$$

In the case of large producers, we consider that each domestic sector has two types of potential competitors. The first type of competitors are small-scale producers, who can produce good η using a technological level A_{St}. Hence it will always be necessary that $p_{Lt} \le p_{St}$, mass production just being feasible at equality. The second type of competitor is a potential industrial competitor with a lower technological level $\chi^{-1} A_{Lt}$, with $\chi > 1$ representing the competitive edge, who is just unwilling to enter the market at zero profit. This competitor also produces on a large scale and supplies the full market. The incumbent will keep to a maximum price level just at the feasibility level for her competitor. We can think that other potential industrial competitors have even lower technologies for the production of this particular good η.

The level of production considered by both the incumbent and her competitor are given by the aggregate expenditure level on this good, $z_t = p_{Lt}(\eta) y_{Lt}(\eta)$, which as we have seen is constant across sectors of all types.

As we see below the maximum markup that the incumbent can use will be χ. Unless we are considering a transition for which mass-production comes into existence, the usual case will be when under the full markup χ nevertheless $p_{Lt} \le p_{St}$. The markup is a measure of the incumbent's market power.

Writing subindex *C* for the incumbent's industrial competitor, since $p_{Ct} y C_t = z_t$, the incumbent will drive the competitor to the zero profit limit, so as in the previous case we will have the corresponding supply determined by $l_{Ct} \chi^{-1} A_{Lt} = \frac{z_t}{p_{Ct}}$, with all income spent on wages, so $w_t l_{Ct} = z_t$. Hence the competitor sets a minimum price $p_{Ct} = \frac{z_t}{l_{Ct} \chi^{-1} A_{Lt}} = \frac{\chi^{-1} w_t}{A_{Lt}}$. At this price

$$p_{Lt} = \frac{\chi w_t}{A_{Lt}}, \quad (8)$$

the incumbent produces the same quantity but employing less labor, $l_{Lt} = \frac{z_t}{p_{Ct} A_{Lt}} = \frac{\chi^{-1} z_t}{w_t}$ at a cost $\chi^{-1} z_t$, hence making a profit

$$\pi_{Lt} = (1 - \chi^{-1}) z_t. \quad (9)$$

2.1.4 The wage level

The wage level can now be obtained by substituting (8) and (6) in (4),

$$1 = p_{Lt}^{\xi} p_{St}^{1-\xi} = \left[\frac{\chi w_t}{A_{Lt}}\right]^{\xi} \left[\frac{w_t}{A_{St}}\right]^{1-\xi}. \quad (10)$$

Hence

$$w_t = \chi^{-\xi} A_{Lt}^{\xi} A_{St}^{1-\xi}. \quad (11)$$

This result directly shows that market power diminishes wages. Substitute (5) back in (6), (8) and simplify to obtain

$$p_{St} = \left[\frac{\chi^{-1} A_{Lt}}{A_{St}}\right]^{\xi}, \quad p_{Lt} = \left[\frac{A_{St}}{\chi^{-1} A_{Lt}}\right]^{1-\xi}. \quad (12)$$

Hence

$$\frac{p_{St}}{p_{Lt}} = \frac{\chi^{-1} A_{Lt}}{A_{St}} > 1. \quad (13)$$

This quantity has to be greater than 1 for large-scale production to outcompete small-scale production and therefore be feasible.

2.1.5 Market clearing for labor

Let the population of the economy be $\mathcal{L}$. Suppose $\mathcal{L}_L$ and $\mathcal{L}_S$ are the aggregate employment levels in sectors

L and *S*, with $\mathcal{L}_L+\mathcal{L}_S=\mathcal{L}$. Then specific sector employment levels l_{Lt}, l_{St} satisfy:

$$\xi l_{Lt}=\mathcal{L}_L,\quad (1-\xi)l_{St}=\mathcal{L}_S,\quad \xi l_{Lt}+(1-\xi)l_{St}=\mathcal{L}. \tag{14}$$

The last equation is the market clearing condition. Now $w_t l_{St}=z_t$, since the participation of labor equals income in sectors *S*, while $w_t l_{Lt}=\chi^{-1}z_t$ in sectors *L*. It follows that

$$\frac{l_{St}}{l_{Lt}}=\chi \tag{15}$$

Hence, we can solve

$$l_{St}=\frac{\mathcal{L}}{\chi^{-1}\xi+(1-\xi)},\quad l_{Lt}=\frac{\chi^{-1}\mathcal{L}}{\chi^{-1}\xi+(1-\xi)}. \tag{16}$$

2.1.6 Income

Note that aggregate income is $Z_t=\int_0^1 z_t d\eta=z_t$. From wages and employment income now follows. Using equation (11) and (16),

$$z_t=w_t l_{St}=\mathrm{Y}A_{Lt}^{\xi}A_{St}^{1-\xi}\mathcal{L}. \tag{17}$$

where $\mathrm{Y}=\frac{\chi^{-\xi}}{\chi^{-1}\xi+(1-\xi)}$.

Note $\frac{d}{d\chi}\left(\chi^{-(1-\xi)}\xi+\chi^{\xi}(1-\xi)\right)=\xi\chi^{\xi-1}\left(1-\chi^{-1}\right)(1-\xi)>0$,

so $\frac{d\mathrm{Y}}{d\chi}<0$. Using (11), the average wage participation is

$$\frac{w_t\mathcal{L}}{z_t}=\chi^{-1}\xi+(1-\xi). \tag{18}$$

As ξ rises, wage participation drops. Wage participation in the large scale sector is lower than in the small scale sector, one reason for Schumacher's "Small is Beautiful" (1973).

2.2 Market power and static efficiency

Following are stated the static distortions due to the presence of market power.

Theorem 1. *Market power distorts the described two sector economy as follows:*

1) *Aggregate income is decreasing in market power.*
2) *The profit to income ratio is increasing in market power.*
3) *Wages and aggregate wage participation are decreasing in market power.*
4) *Employment intensity l_{Lt} in the large scale sector is decreasing in market power, while employment intensity l_{St} in the small scale sector is increasing in market power.*

Proof. 1) See (17) and the proof below showing $\frac{d\mathrm{Y}}{d\chi}<0$. 2) See (9). 3) See (11). 4) See (16).

2.2.1 Wages, market power and the size of the large scale sector

Let us examine how the size of the large scale sector ξ affect wages in the presence of market power χ.

Theorem 2. *When the size of the large scale sector increases, wages respond as follows:*

$$\frac{\partial \ln w_t}{\partial \xi}=\frac{\partial}{\partial \xi}\left(-\xi\ln\chi+\xi\ln A_{Lt}+(1-\xi)A_{St}\right)=\ln\frac{A_{Lt}}{\chi A_{St}}\geq 0.$$

Proof. Differentiate (11) and note (13).

Note that the impact of industrialization on wages can be low if market power is near its maximum feasible level $\chi=\frac{A_{Lt}}{A_{St}}$, when specific large scale sectors face low large scale competition. Furthermore, if no small scale competition is faced either, the impact of industrialization on wages could be negative.

2.3 Technological change

I now consider a two country model and define a process of endogenous change for the technological levels A_{Lt}, A_{St} of one of the countries. This framework can be extended in further work to the context of trade and FDI.

As in MF and Mayer-Foulkes (2015b), I consider a myopic decision maker who has perfect foresight as her time horizon Δt tends to zero. This is both more realistic (there *is* no perfect foresight!) and simpler. It eliminates the need for a second set of variables predicting the prices of all goods (forever!) that is required in perfect foresight models. In addition, scale effects occurring due to future relative sectorial sizes affecting innovator's incentives are brought to the present.

Mayer-Foulkes (2015b) shows that perfect foresight as $\Delta t \to 0$ is equivalent to defining perfect myopic foresight as having perfect knowledge of the current economic variables' time derivatives. The myopic agent uses this knowledge to maximize the current time derivative of profits.

2.3.1 Innovation in the large scale sector

As mentioned above, there is in each mass production sector a single, infinitely lived innovator who can produce an innovation for the next period. The innovator has perfect myopic foresight and maximizes the current time derivative of profits.

The effectiveness of innovation investment of the sector η entrepreneur has two components. The first is a material input flow v_{Lt}. The second is knowledge and is proportional to two components. 1) The skill level $S_{Lt}=A_{Lt}$ that the typical firm has been able to accumulate in production, which we assume is the technological level of her firm, analogously to Howitt and Mayer-Foulkes (2005). This generates a disadvantage of backwardness. 2) To a knowledge externality proportional to the difference between the nascent leading technological edge in Country 1, $(1+\sigma_{1L})\,A_{1Lt}$ and the current technological level A_{Lt}. Here A_{1Lt} is the leading technological edge, which is the technological level of the large scale sector in Country 1. The term $(1+\sigma_{1L})\,A_{1Lt}$ represents the nascent technological possibilities implicit at this technological level. Examples are the potential use of other firm's new embodied technologies at time $t+\Delta t$, or of their new ideas. The difference with A_{Lt} measures how far back our innovating firm, situated in a leading or a lagging country, is from these nascent possibilities. This generates an advantage of backwardness. In Country 1 the knowledge externalities will be proportional to $\sigma_{1L}A_{1Lt}$, while in Country 2 they will be larger, proportional to $\sigma_{1L}A_{1Lt}+A_{1Lt}-A_{2Lt}$. Now, due to the fishing out effect, the effectiveness of this knowledge externality is defined to be inversely proportional to the leading technological level A_{1Lt}. Innovation occurs with certainty combining these material and knowledge components to obtain a rate of change of the technological level at time t given by:

$$\frac{\partial}{\partial \Delta t}\tilde{A}_L\left(t+\Delta t, v_{Lt+\Delta t}\right)\Big|_{\Delta t=0} = \mu_L\left(\left(\frac{(1+\sigma_{1L})A_{1Lt}-A_{Lt}}{A_{1Lt}}\right)S_{Lt}\right)^{\upsilon} v_{Lt}^{1-\upsilon}, \quad (20)$$

where $\mu_L, \sigma_{1L} > 0,\ 0 < \upsilon < 1$. Here $\tilde{A}_{Lt+\Delta t} \equiv \tilde{A}_L\left(t+\Delta t, v_{Lt+\Delta t}\right)$ is a technology trajectory envisaged by the incumbent over a small time interval $[t, t+\Delta t)$ into the future, given an expenditure level $v_{Lt+\Delta t}$ on innovation. Note that at $\Delta t = 0,\ \tilde{A}_L\left(t, v_{Lt}\right) = A_{Lt}$. In particular $\frac{\partial}{\partial v_{Lt}}\tilde{A}_L\left(t, v_{Lt}\right) = 0$.

The parameter μL represents the innovation productivity of the combined inputs, which may differ in the two countries.

The incumbent's mark up, at time $t+\Delta t$ will be $\frac{\chi\tilde{A}_{Lt+\Delta t}}{A_{Lt+\Delta t}}$. Thus, using myopic perfect foresight, at any given time t she maximizes her expected rate of change of profit

$$\max_{v_{Lt}}\left[\frac{d}{d\Delta t}\left[(1-\phi_L^{\pi})\left(1-\left(\frac{\chi\tilde{A}_{Lt+\Delta t}}{A_{Lt+\Delta t}}\right)^{-1}\right)z_{t+\Delta t}\right]\Bigg|_{\Delta t=0} - (1-\phi_L^{\iota})\,v_{Lt}\right]. \quad (21)$$

where $\phi_L^{\pi}, \phi_L^{\iota} \in (0,1)$ represent a profit tax and an innovation subsidy for the large scale sector, positive or negative proxies for all distortions and policies affecting profits and the incentives to innovate. The first order condition is:

$$0 = \frac{\partial}{\partial v_{Lt}}\left[\frac{d}{d\Delta t}\left[(1-\phi_L^{\pi})\left(1-\left(\frac{\chi\tilde{A}_{Lt+\Delta t}}{A_{Lt+\Delta t}}\right)^{-1}\right)z_{t+\Delta t}\right]\Bigg|_{\Delta t=0} - (1-\phi_L^{\iota})\,v_{Lt}\right] \quad (22)$$

$$= (1-\phi_L^{\pi})\left(\frac{\chi\bar{A}_{Lt}}{A_{Lt}}\right)^{-2}\frac{\chi\frac{\partial}{\partial v_{Lt}}\frac{d}{d\Delta t}\tilde{A}_L\left(t+\Delta t, v_{Lt+\Delta t}\right)\big|_{\Delta t=0}}{A_{Lt}}z_t - (1-\phi_L^{\iota}), \quad (23)$$

since all other terms are zero. Substituting (20) and simplifying,

$$0=\left(1-\phi_L^{\pi}\right)(1-\upsilon)\mu_L\left(\frac{(1+\sigma_{1L})A_{1Lt}-A_{Lt}}{A_{1Lt}}\right)^{\upsilon}S_{Lt}^{\upsilon}\upsilon_{Lt}^{-\upsilon}\frac{z_t}{\chi A_{Lt}}-\left(1-\phi_L^{\iota}\right). \quad (24)$$

Letting $\hat{\mu}_L=\frac{(1-\upsilon)\left(1-\phi_L^{\pi}\right)}{\left(1-\phi_L^{\iota}\right)}\mu_L$, material inputs v_{Lt} are given by:

$$\upsilon_{Lt}=\left(\frac{\hat{\mu}_L}{\chi}\frac{z_t}{A_L}\right)^{\frac{1}{\upsilon}}\left(\frac{(1+\sigma_{1L})A_{1Lt}-A_{Lt}}{A_{1Lt}}\right)S_{Lt}, \quad (25)$$

Substituting this result in (20),

$$\frac{\partial}{\partial\Delta t}\tilde{A}_L\left(t+\Delta t,\upsilon_{Lt+\Delta t}\right)\Big|_{\Delta t=0}=\mu_L\left(\frac{(1+\sigma_{1L})A_{1Lt}-A_{Lt}}{A_{1Lt}}\right)S_{Lt}\left(\frac{\hat{\mu}_L}{\chi}\frac{z_t}{A_L}\right)^{\frac{1-\upsilon}{\upsilon}}. \quad (26)$$

Note that since z_t depends on both A_{Lt} and A_{St}, a relative scale effects that complicates the dynamics once technological change in both variables is considered. This aspect is simplified by using continuous myopic foresight. Note also that innovation is *decreasing* in market power χ, because, as can be seen by following the derivation above, the higher the market power, the relatively lower costs are compared to profits and therefore the lower the impact of the cost of technological improvement on profits. The easier it is to make profits, the relatively less it is worth to spend on cost-saving innovation.

Note now that perfect myopic foresight implies $\frac{\partial}{\partial\Delta t}\tilde{A}_L\left(t+\Delta t,\upsilon_{Lt}\right)\Big|_{\Delta t=0}=\frac{d}{dt}A_{Lt}$. Substituting $S_{Lt}=A_{Lt}$ and setting $\varsigma=\frac{1-\upsilon}{\upsilon}$,

$$\tilde{\mu}_L=\mu_L\hat{\mu}_L^{\varsigma}=\left(\frac{(1-\upsilon)\left(1-\phi_L^{\pi}\right)}{1-\phi_L^{\iota}}\right)^{\varsigma}\mu_L^{1+\varsigma}, \quad (27)$$

$$\frac{d}{dt}\ln A_{Lt}=\tilde{\mu}_L\left(\frac{(1+\sigma_{1L})A_{1Lt}-A_{Lt}}{A_{1Lt}}\right)\left(\frac{z_t}{\chi A_{Lt}}\right)^{\varsigma}. \quad (28)$$

2.3.2 Innovation in the small scale sector

In the case of the small scale sector I assume that each competitive producer can innovate to reap the productive benefits of new technologies. Each entrepreneur has perfect myopic foresight and maximizes the current time derivative of profits. As before, the effectiveness of innovation investment has two components. The first is a material input flow v_{St}. The second is knowledge and is proportional to two components. 1) The skill level S_{St} of the firm (entrepreneur, workers and installed productivity), equal to A_{St}. This generates a disadvantage of backwardness, reflecting the limits to technological change imposed by ongoing backwardness. 2) To a knowledge externality proportional to the difference between the technological level A_{Lt} of the large scale sector and the current small scale technological level A_{St}. Thus we assume for simplicity that leading edge technologies have first to be adopted in a domestic industrial context to become useful in the small scale context. For example, cell phones cannot be used without a telephone company. Machines cannot only be purchased from a domestic producer, since there is no trade or foreign direct investment, and so on. The knowledge externality is proportional to $\frac{A_{Lt}-A_{St}}{A_{Lt}}$. In this setting the small scale sector always lag behind the large scale sector. Hence the numerator generates an advantage of backwardness. The denominator represents the fishing out effect. Innovation occurs with certainty combining the material and knowledge components to obtain a rate of change of the technological level at time *t* given by:

$$\frac{\partial}{\partial\Delta t}\tilde{A}_S\left(t+\Delta t,\upsilon_{St+\Delta t}\right)\Big|_{\Delta t=0}=\mu_S\left(\frac{A_{Lt}-A_{St}}{A_{Lt}}S_{St}\right)^{\upsilon}\upsilon_{St}^{1-\upsilon}. \quad (29)$$

Here $\tilde{A}_{St+\Delta t}\equiv\tilde{A}_S\left(t+\Delta t,\upsilon_{St+\Delta t}\right)$ is the technology trajectory envisaged by the entrepreneur over a small time interval $[t,t+\Delta t)$ into the future. The innovation productivity μ_S is analogous to μ_L, except that it reflects a limited kind of innovation, the kind of innovation that can be carried out on a small rather than large scale $\mu_S<\mu_L$. This is analogous to the distinction between implementation and R&D in Howitt and Mayer-Foulkes (2005), in that in the small scale innovation is unlikely to use an R&D lab, employ scientists, and so on, and is more likely simply to implement technologies created in the large scale sector.

Recall that the defining characteristic of the small scale sector is that it cannot obtain sufficient profit margins over significant market sectors. For simplicity we assume all small scale firms are symmetric and know that the others will behave symmetrically. The firms are the same size, and this size is limited by a maximum level of sales $\bar{z}_t$, much smaller than the aggregate sales level z_t in her market. One way to think about this size is

$\bar{z}_t = \frac{z_t}{N}$, which divides the market into some large

number of firms N representing an approximation to perfect competition.

Small scale innovation (29) is now analogous to large

scale innovation (20) except that $\mu_L \left(\frac{(1+\sigma_{1L})A_{1Lt}-A_{Lt}}{A_{1Lt}}\right)^{\upsilon}$

becomes $\mu_S \left(\frac{A_{Lt}-A_{St}}{A_{Lt}}\right)^{\upsilon}$, z_t becomes $\bar{z}_t = \frac{z_t}{N}$, and χ becomes 1. We consider an innovation subsidy $\phi_S^{\iota} \in (0,1)$, but not a profit tax φ_S^{π}, since there is no profit. Hence the same derivation yields material inputs given by:

$$v_S = (\hat{\mu}_S z_t)^{\frac{1}{\upsilon}} \left(\frac{A_{Lt}-A_{St}}{A_{Lt}}\right) A_{St}^{-\varsigma}. \quad (30)$$

where $\hat{\mu}_S = \frac{1-\upsilon}{N(1-\phi_S^{\iota})}\mu_S$, and therefore

$$\frac{d}{dt}\ln A_{St} = \tilde{\mu}_S \frac{A_{Lt}-A_{St}}{A_{Lt}} \left(\frac{z_t}{A_{St}}\right)^{\varsigma}, \quad (31)$$

with $\tilde{\mu}_S = \left(\frac{1-\upsilon}{N(1-\phi_S^{\iota})}\right)^{\varsigma} \mu_S^{1+\varsigma}$.

2.3.3 The technological dynamics

Write A_{iLt}, A_{iSt}, i=1,2 for the technological levels of the large and small scale sectors in Countries 1 and 2.

Definition 3*. Define the relative technological levels*

$$a_{it} = \frac{A_{iSt}}{A_{iLt}}, \quad b_t = \frac{A_{2Lt}}{A_{1Lt}}, \quad i=1,2. \quad (32)$$

We can write (17) in the form

$$\frac{z_{it}}{A_{iLt}} = \Upsilon_i a_{it}^{1-\xi}\mathcal{L}, \quad (33)$$

where $\Upsilon_i = \frac{\chi_i^{-\xi}}{\chi_i^{-1}\xi+(1-\xi)}$. Substituting in (28), (31),

$$\frac{d}{dt}\ln A_{iSt} = \tilde{\mu}_{iS}(1-a_{it})\left(\Upsilon_i a_{it}^{1-\xi}\mathcal{L}_i\right)^{\varsigma}, \quad (34)$$

$$\frac{d}{dt}\ln A_{1Lt} = \tilde{\mu}_{1L}\chi_1^{-\varsigma}\sigma_{1L}\left(\Upsilon_1 a_{1t}^{1-\xi}\mathcal{L}_1\right)^{\varsigma}, \quad (35)$$

$$\frac{d}{dt}\ln A_{2Lt} = \tilde{\mu}_{2L}\chi_2^{-\varsigma}(1+\sigma_{1L}-b_t)\left(\Upsilon_2 a_{2t}^{1-\xi}\mathcal{L}_2\right)^{\varsigma}. \quad (36)$$

Each of these equations displays the effect of market size, knowledge externality, innovation productivity and market power. In effect, as we have commented above, market power reduces the productivity of innovation in the large scale sector, because it reduces the relative impact of cost reduction. Thus we can define for reference below:

Definition 3. *The effective innovation productivity in each large scale sector is* $\tilde{\mu}_{iL}\chi_i^{-\varsigma}$*, for Countries i=1,2.*

We can now turn to the dynamics of the relative technological levels.

$$\frac{d}{dt}\ln a_{1t} = \left[\tilde{\mu}_{1S}(1-a_{1t}) - \tilde{\mu}_{1L}\sigma_{1L}\chi_1^{-\varsigma}\right]\left(\Upsilon_1 a_{1t}^{1-\xi}\mathcal{L}_1\right)^{\varsigma}, \quad (37)$$

$$\frac{d}{dt}\ln a_{2t} = \left[\tilde{\mu}_{2S}(1-a_{2t}) - \tilde{\mu}_{2L}(1+\sigma_{1L}-b_t)\chi_2^{-\varsigma}\right]\left(\Upsilon_2 a_{2t}^{1-\xi}\mathcal{L}_2\right)^{\varsigma}, \quad (38)$$

$$\frac{d}{dt}\ln b_t = \tilde{\mu}_{2L}(1+\sigma_{1L}-b_t)\chi_2^{-\varsigma}\left(\Upsilon_2 a_{2t}^{1-\xi}\mathcal{L}_2\right)^{\varsigma} - \tilde{\mu}_{1L}\sigma_{1L}\chi_1^{-\varsigma}\left(\Upsilon_1 a_{1t}^{1-\xi}\mathcal{L}_1\right)^{\varsigma}. \quad (39)$$

The right hand sides of these three equations define the functions $H_1(a_{1t})$, $H_2(a_{2t}, b_t)$, $H_b(a_{1t}, a_{2t}, b_t)$.

2.3.4 Steady state in the leading country

The dynamics (37) of the relative technological level between the small and large scale sectors in leading Country 1 are independent. Note that $H_1(1) < 0$. This means the small scale sector of the leading country cannot overtake the large scale sector in technological level.

Theorem 3. *The relative technological level a_{1t} of the small to the large scale sector in Country 1 has a unique stable positive steady state $a_1^* \in (0,1)$. The steady state growth*

rate is $\gamma = \sigma_{1L}\tilde{\mu}_L \left(\frac{\Upsilon_1}{\chi_1} a_1^{*(1-\xi)} \mathcal{L}_1\right)^{\varsigma}$, which is increasing in a_1^* and decreasing in χ_1.

Proof. Note that $H_1(a_{1t})=0$ has two solutions, 0 and $a_1^* = 1 - \frac{\bar{\mu}_{1L}\sigma_{1L}}{\chi^{\varsigma}\bar{\mu}_{1S}}$. H_1 is negative above and positive below a_1^*, so the steady state at 0 is unstable and a_1^* is the unique stable steady state. The growth rate $\gamma = \frac{d}{dt}\ln A_{1Lt} = \sigma_{1L}\tilde{\mu}_L \left(\frac{\Upsilon_1}{\chi_1} a_1^{*(1-\xi)} \mathcal{L}_1\right)^{\varsigma}$. This is decreasing in χ_1 since $\frac{\Upsilon_1}{\chi_1} = \frac{\chi_1^{-\xi}}{\xi+\chi_1(1-\xi)}$ is decreasing in χ_1.

The scale effect in the growth rate is partly counteracted by innovation dedicated to variety, which we do not address here.

2.3.5 Steady state in the lagging country

In the case of lagging Country 2 the technological dynamics (38), (39) involve two additional variables a_{2t} and b_t, the relative level of the small scale to the large scale sector in Country 2, and the relative level of Country 2's large scale sector to leading Country 1's large scale sector.

Note that $H_2(1,b_t)<0$, so Country 2's small scale sector cannot catch up with its large scale sector. I assume that when leading Country 1 is near its steady state Country 2's large scale sector cannot catch up with Country 1's large scale sector, so $H_b\left(a_1^*, a_{2t}, 1\right)<0$, that is,

$$\tilde{\mu}_{2L}\chi_2^{-\varsigma}\left(\Upsilon_2 a_{2t}^{1-\xi}\mathcal{L}_2\right)^{\varsigma} < \tilde{\mu}_{1L}\chi_1^{-\varsigma}\left(\Upsilon_1 a_1^{*(1-\xi)}\mathcal{L}_1\right)^{\varsigma}. \quad (40)$$

To understand the technological dynamics we examine the phase diagram of dynamical system (38), (39), on square subset $S=[0,1]\times[0,1]$ of the (a_{2t},b_t) plane. Let $L_{a_{2t}}$ and L_{b_t} be the subsets of the square for which $\frac{d}{dt}\ln a_{2t}=0$ and $\frac{d}{dt}\ln b_t=0$ respectively.

$L_{a_{2t}}$ is on a curve $b_t = f^a(a_{2t})$ given by

$$b_t = 1 + \sigma_{1L} - \frac{\chi_2^{\varsigma}\tilde{\mu}_{2S}(1-a_{2t})}{\tilde{\mu}_{2L}}, \quad (41)$$

a line with positive slope. In addition

$\frac{\partial}{\partial b_t}\frac{d\ln a_{2t}}{dt} = \tilde{\mu}_{2L}\chi_2^{-\varsigma}\left(\Upsilon_2 a_{2t}^{1-\xi}\mathcal{L}_2\right)^{\varsigma} > 0$, so above the line $\frac{d\ln a_t}{dt}>0$ and below the line $\frac{d\ln a_t}{dt}<0$. Since we assume that the small scale sector cannot overtake the large scale sector, $f^a(1)>1$.

L_{b_t} is on a curve $b_t = f^b(a_{2t})$ given by

$$b_t = 1 + \sigma_{1L} - \frac{\tilde{\mu}_{1L}\sigma_{1L}\chi_1^{-\varsigma}\left(\Upsilon_1 a_1^{*(1-\xi)}\mathcal{L}_1\right)^{\varsigma}}{\tilde{\mu}_{2L}\chi_2^{-\varsigma}\left(\Upsilon_2 a_{2t}^{1-\xi}\mathcal{L}_2\right)^{\varsigma}}. \quad (42)$$

$f^b(a_t)$ is increasing and concave since $f^{b\prime}(a_{2t})>0$ and $f^{b\prime\prime}(a_{2t})<0$. Note that there is a value $a_2^b>0$ for which $f^b\left(a_2^b\right)=0$. Next, note that $\frac{\partial}{\partial b_t}\frac{d\ln b_t}{dt} = -\tilde{\mu}_{2L}\chi_2^{-\varsigma}\left(\Upsilon_2 a_{2t}^{1-\xi}\mathcal{L}_2\right)^{\varsigma}<0$, so $\frac{d\ln b_t}{dt}>0$ for $b_t<f^b\left(a_2^b\right)$ and $\frac{d\ln b_t}{dt}<0$ for $b_t>f^b\left(a_2^b\right)$.

Hence, since Country 2's large scale sector cannot overtake Country 1's, $f^b(1)\le 1$.

There are four ways in which the segment $L_{a_{2t}}$ can be positioned with respect to the curve L_{b_t}, illustrated in the four panels of Figure 1.

Note that $\frac{d}{dt}\ln a_{2t}$ well defined for $a_{2t}>0$, even on the a_{2t} axis. Let

$$a_2^{**} = \max\left(1 - \frac{\tilde{\mu}_{2L}(1+\sigma_{1L})}{\chi_2^{\varsigma}\tilde{\mu}_{2S}}, 0\right). \quad (43)$$

The first term is the value of a_{2t} at which $L_{a_{2t}}$ crosses the a_{2t} axis. Suppose $a_2^{**}<a_2^b$. Then, as in Panels 2, 3 and 4, there is a non-empty set $\left\{(a_{2t},0): a_2^{**}\le a_{2t}\le a_2^b\right\}$ on the

a_{2t} axis for which $\frac{d\ln b_t}{dt}<0$ and $\frac{d\ln a_{2t}}{dt}<0$. Correspondingly, define $b^{**}=0$. $\left(a_2^{**},b^{**}\right)$ represents a stable steady state. At these steady states, $\frac{d\ln b_t}{dt}<0$ represents the amount by which the large scale sector in Country 2 is growing less than Country 1's. If also if

$a_2^{**}=0,\ \frac{d}{dt}\ln a_{2t}=\tilde{\mu}_{2S}-\tilde{\mu}_{2L}\left(1+\sigma_{1L}\right)\chi_2^{-\varsigma}$ epresents

the amount by which the small scale sector is growing less than the large scale sector in Country 2. In the special case $a_2^{**}=a_2^b$, $\frac{d\ln b_t}{dt}=0$ so there is no divergence in growth rates.

Note that there is never a steady state on the b_t axis with $b_t>0$ since here

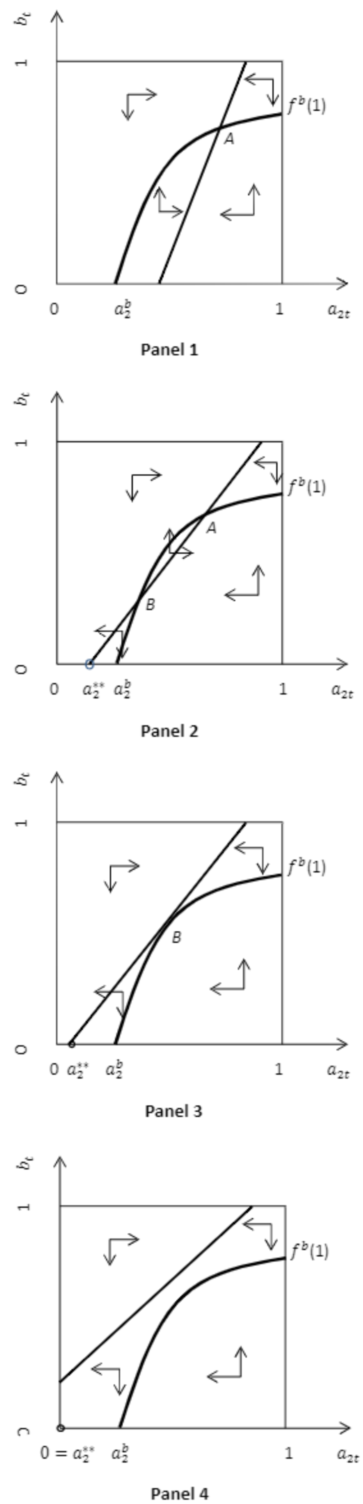
$$\frac{d}{dt}\ln b_t=-\tilde{\mu}_{1L}\sigma_{1L}\chi_1^{-\varsigma}\left(\Upsilon_1 a_1^{*(1-\xi)}\mathcal{L}_1\right)^{\varsigma}<0. \quad (44)$$

Summarizing,

Theorem 4. *Suppose the leading country is at a steady state $a_1^*\in(0,1)$. Then there are four configurations of the phase diagram, according to whether the loci La_{2t} and L_{b_t} intersect once non-tangentially, twice, once tangentially, or do not intersect at all. An intersection $\left(a_2^*,b^*\right)$ for which $f^{a\prime}\left(a_2^*\right)>f^{b\prime}\left(a_2^*\right)$ defines a stable steady state, labeled A in Figure 1. An intersection $\left(a_2^{\#},b^{\#}\right)$ for which $f^{a\prime}\left(a_2^*\right)\leq f^{b\prime}\left(a_2^*\right)$ defines an unstable steady state, labeled B in Figure 1. When $a_2^{**}\leq a_2^b$, this is a stable steady state on the a_{2t} axis, with corresponding $b^{**}=0$, for which if $a_2^{**}<a_2^b$ the large scale sector technological level grows slower in Country 1 than in Country 2. If also $a_2^{**}=0$, the small scale sector is growing less than the large scale sector in Country 2.*

For steady states of type A, when $\tilde{\mu}_{2S}$ increases, both a_2^* and b^* increase. When χ_2 decreases or $\tilde{\mu}_{2L}$ increases, b increases and a_2^* remains constant. For steady states of type B, a_2^{**} is increasing in $\tilde{\mu}_{2S}$ and χ_2, and decreasing in $\tilde{\mu}_{2L}$. In particular the growth rates of*

Figure 1. Phase Diagram configurations for lagging Country 2.
In each case A is a stable steady state, B is an unstable steady state. There are also stable steady states $(a_2^{**},0)$ (of type C, marked with a small circle) at which the whole economy diverges in growth rates, and possibly also the small scale sector.



both sectors are increasing in $\tilde{\mu}_{2S}$ at steady state a_2^{**}, while the small sector's growth rate can be positively or negatively affected by increases in $\tilde{\mu}_{2L}$. For steady states of type C, both growth rates tends to zero. However, near the steady state the growth rate of the small scale sector is increasing in $\tilde{\mu}_{2S}$ the large scale sector remaining unaffected; the opposite holding temporarily with $\tilde{\mu}_{2L}$.

Proof. Only the comparative statics remain to be proved. For steady states of type A, note that loci $L_{a_{2t}}$, L_{b_t} given by (41), (42) can both be written in the form

$(1+\sigma_{1L}-b_t)=\frac{\chi_2^{\varsigma}}{\tilde{\mu}_{2L}}h(a_{2t})$, with a positive functions *h*.

Hence when χ_2 increases a_2^* remains constant while b^* decreases. Similarly, when $\tilde{\mu}_{2L}$ increases a_2^* remains constant while b^* increases. To obtain the static analysis when $\tilde{\mu}_{2S}$ varies, write the locus $L_{a_{2t}}$ as $b_t=f^a(a_{2t},\tilde{\mu}_{2S})$, the locus L_{b_t} remaining as $b_t=f^b(a_{2t})$. Note $\frac{\partial f^a}{\partial\tilde{\mu}_{2S}}<0$. Hence

$$\frac{\partial b^*}{\partial\tilde{\mu}_{2S}}=\frac{\partial f^a}{\partial a_{2t}}\frac{\partial a_{2t}}{\partial\tilde{\mu}_{2S}}+\frac{\partial f^a}{\partial\tilde{\mu}_{2S}},\qquad\frac{\partial b^*}{\partial\tilde{\mu}_{2S}}=\frac{\partial f^b}{\partial a_{2t}}\frac{\partial a_{2t}}{\partial\tilde{\mu}_{2S}}. \quad (45)$$

Since $f^{a\prime}(a_2^*)>f^{b\prime}(a_2^*)$ at type A steady states it follows that

$$\frac{\partial a_2^*}{\partial\tilde{\mu}_{2S}}=\frac{-\frac{\partial f^a}{\partial\tilde{\mu}_{2S}}}{\frac{\partial f^a}{\partial a_{2t}}-\frac{\partial f^b}{\partial a_{2t}}}>0,\qquad\frac{\partial b^*}{\partial\tilde{\mu}_{2S}}>0. \quad (46)$$

For steady states of type B, the results follow from expression $a_2^{**}=1-\frac{\tilde{\mu}_{2L}(1+\sigma_{1L})}{\chi_2^{\varsigma}\tilde{\mu}_{2S}}$, see equation (43).

Note that at these steady states the growth rate is:

$$\gamma_S^{**}=\frac{d}{dt}\ln(a_{2t}b_t)=\tilde{\mu}_{2S}(1-a_2^{**})\left(\Upsilon_2a_2^{**(1-\xi)}\mathcal{L}_2\right)^{\varsigma}-\tilde{\mu}_{1L}\sigma_{1L}\chi_1^{-\varsigma}\left(\Upsilon_1a_1^{*(1-\xi)}\mathcal{L}_1\right)^{\varsigma}. \quad (47)$$

$$\gamma_L^{**}=\tilde{\mu}_{2L}(1+\sigma_{1L})\chi_2^{-\varsigma}\left(\Upsilon_2a_2^{**(1-\xi)}\mathcal{L}_2\right)^{\varsigma}-\tilde{\mu}_{1L}\sigma_{1L}\chi_1^{-\varsigma}\left(\Upsilon_1a_1^{**(1-\xi)}\mathcal{L}_1\right)^{\varsigma} \quad (48)$$

Since $\frac{\partial a_2^{**}}{\partial\tilde{\mu}_{2S}}=\frac{\tilde{\mu}_{2L}(1+\sigma_{1L})}{\chi_2^{\varsigma}\tilde{\mu}_{2S}^2}$, it follows that

$\frac{\partial}{\partial\tilde{\mu}_{2S}}\left[\tilde{\mu}_{2S}(1-a_2^{**})\right]=(1-a_2^{**})-\tilde{\mu}_{2S}\frac{\tilde{\mu}_{2L}(1+\sigma_{1L})}{\chi_2^{\varsigma}\tilde{\mu}_{2S}^2}=0$. Hence

$\frac{\partial\gamma_S^{**}}{\partial\tilde{\mu}_{2S}}>0$. Also, since $\frac{\partial a_2^{**}}{\partial\tilde{\mu}_{2S}}>0$, it follows $\frac{\partial\gamma_L^{**}}{\partial\tilde{\mu}_{2S}}>0$.

The impact of $\tilde{\mu}_{2L}$ on γ_S^{**} is positive for small ς and negative for large ς, when the disadvantage of backwardness becomes relatively larger.

For steady states of type C, a_2^{**} is zero independently of small changes in $\tilde{\mu}_{2S}$. Therefore so is the growth rate γ_S^{**}, still given by (47). Therefore, near the steady state the growth rate is still increasing in $\tilde{\mu}_{2S}$, while γ_L^{**} is independent of $\tilde{\mu}_{2S}$.

2.4 Deeper backwardness in Country 2

A general feature of underdevelopment can be proved: In underdeveloped countries the small scale sector lags further behind the large scale sector than in developed countries.

Theorem 5. *Suppose Countries 2 cannot catch up with Country 1, and the relative effective innovation productivity ratio between the large and small scale sectors in Countries 1 and 2 are equal, that is,* $\frac{\tilde{\mu}_{2L}\chi_2^{-\varsigma}}{\tilde{\mu}_{2S}}=\frac{\tilde{\mu}_{1L}\chi_1^{-\varsigma}}{\tilde{\mu}_{1S}}$.

Then $a_2^*<a_1^*$: *the small scale sector lags further behind the large scale sector in Country 2 than in Country 1.*

Proof. The locus $L_{a_{2t}}$ meets the b_t=1 line at

$a_{2t}=a_2^{b=1}\equiv1-\frac{\tilde{\mu}_{2L}\chi_2^{-\varsigma}\sigma_{1L}}{\tilde{\mu}_{2S}}$. Hence, since $L_{a_{2t}}$ is positively sloped,

$$a_1^*-a_2^*>a_1^*-a_2^{b=1}=\sigma_{1L}\left(\frac{\tilde{\mu}_{2L}\chi_2^{-\varsigma}}{\tilde{\mu}_{2S}}-\frac{\tilde{\mu}_{1L}\chi_1^{-\varsigma}}{\tilde{\mu}_{1S}}\right)=0. \quad (49)$$

An alternative proof is the following. At the steady states, equations (37), (38), (39) imply

$$\tilde{\mu}_{1S}(1-a_1^*)\left(\Upsilon_1a_1^{*(1-\xi)}\mathcal{L}_1\right)^{\varsigma}=\tilde{\mu}_{1L}\sigma_{1L}\chi_1^{-\varsigma}\left(\Upsilon_1a_1^{*(1-\xi)}\mathcal{L}_1\right)^{\varsigma} \quad (50)$$

$$=\tilde{\mu}_{2L}(1+\sigma_{1L}-b^*)\chi_2^{-\varsigma}\left(\Upsilon_2a_2^{*(1-\xi)}\mathcal{L}_2\right)^{\varsigma} \quad (51)$$

$$=\tilde{\mu}_{2S}(1-a_2^*)\left(\Upsilon_2a_2^{*(1-\xi)}\mathcal{L}_2\right)^{\varsigma}. \quad (52)$$

Hence, given the hypothesis and (40),

$$\frac{1-a_1^*}{1-a_2^*}=\frac{\tilde{\mu}_{2L}\chi_2^{-\varsigma}\tilde{\mu}_{1S}\left(1-a_1^*\right)}{\tilde{\mu}_{1L}\chi_1^{-\varsigma}\tilde{\mu}_{2S}\left(1-a_2^*\right)}=\frac{\tilde{\mu}_{2L}\chi_2^{-\varsigma}\left(\Upsilon_2 a_2^{*(1-\xi)}\mathcal{L}_2\right)^{\varsigma}}{\tilde{\mu}_{1L}\chi_1^{-\varsigma}\left(\Upsilon_1 a_1^{*(1-\xi)}\mathcal{L}_1\right)^{\varsigma}}>1. \quad (53)$$

2.5 Government incentives for innovation

Can the government improve on the private assignment of innovation resources by subsidizing innovation? If so, can it pay for this by taxing profits?

Following MF, in accordance with perfect myopic foresight, let the government maximize the growth rate of aggregate income Z_t net of the flow of resources dedicated to innovation,

$$\max_{v_{Lt},v_{St}} \frac{\partial}{\partial \Delta t}\tilde{Z}\left(t+\Delta t, v_{Lt+\Delta t}, v_{St+\Delta t}\right)\Big|_{\Delta t=0}-\left[\xi v_{Lt}+\left(1-\xi\right)N v_{St}\right]. \quad (54)$$

This optimization assumes market exchange takes place in the presence of market power, so the question posed is only seeking a second best. Here $\tilde{Z}\left(t+\Delta t, v_{Lt+\Delta t}, v_{St+\Delta t}\right)$ is an income trajectory envisaged by the government over a small time interval into the future, given expenditure levels v_{Lt} in innovation investment in each large scale sector, and v_{St} in innovation investment by each of the N firms in each small scale sector. The maximization is subject to the physical equations for technological change (20) and (29). Note that the N small firms still repeat innovation in this government maximization. The following efficiency results are obtained.

Theorem 6. *In the large scale sector,*

1) *As market power tends to zero, when $\chi\to 1$, privately assigned innovation tends to efficiency.*
2) *When the market power tax is applied, as $x0\to 1$, case 1) is approached in the limit.*
3) *Suppose that large scale sector profits are quantitatively higher than the shortfall for optimal innovation investment. Then taxes and subsidies φ_L^{π}, $\varphi_L^{\iota}\in(0,1)$ exist for which the government's budget is balanced and innovation is optimal. If profits are not that high, a lump sum tax on wages is needed to obtain optimal innovation with a balanced budget.*

Proof. Substituting $\mu_L\left(\frac{(1+\sigma_{1L})A_{1Lt}-A_{Lt}}{A_{1Lt}}\right)^{\upsilon}$ for μ_L,

the same proof holds as in MF.

3 Conclusions

The model developed here analyses the distribution and efficiency properties of a developed or underdeveloped industrial market economy as a function of the dynamics of innovation and absorption. A large scale production sector innovates for market power, while a small scale, competitive production sector absorbs the technologies created in the innovative sector. The following results have been proved.

First, the static results. Theorem 1 shows that aggregate net income is decreasing in the large scale sector market power, because higher prices for mass produced goods divert resources from inputs to profits. It also shows aggregate profits and profits per sector are increasing in market power, while wages and aggregate wage participation are decreasing in market power. The aggregate wage to profit ratio is decreasing in market power and in the number of large-scale sectors. Employment intensity in the large scale sector is decreasing in market power, the opposite holding for the small scale sector.

Theorem 2 shows the impact on wages of an increase in the number of large scale sectors is decreasing in market power. When market power is high enough, wages can remain unaffected by the technological level of the large scale sector.

Now, the dynamics of the model, generated by technological change. To simplify these, we use the concept of perfect myopic foresight into the infinitesimal future. This is equivalent to perfect foresight on the current time derivatives of relative variables, for example when maximizing the rate of change of profits. Innovation inputs are: the current state of knowledge, including installed technology, and material inputs. We assume the incumbent has a small innovation advantage. The lagging country's large scale sector benefits from absorbing technologies from the leading country's large scale sector. Each country's small scale sector absorbs technologies from the large scale sector. The optimal private rates of investment in technological change for the large and small scale sectors can now be derived. These result in the rates of technological change for both sectors — the rates of innovation and absorption.

Theorem 3 shows, as in MF, that there is a steady state for the relative technological level between the small and large scale sectors in leading Country 1. The corresponding steady state growth rate is increasing in the relative level of the small scale sector and decreasing in the market power of the large scale sector.

Two variables describe the steady state of lagging Country 2, the relative level b_t of its large scale sector to Country 1's, and the relative level a_{2t} between its small and its large scale sectors. Theorem 4 shows both variables a_2^* and b^* may have a positive steady state, or b^* may be zero, or both may be zero. The positive cases represent parallel growth, and the zero cases represent parallel growth not achieved: divergence in growth rates.

Let us turn to the comparative statics for the lagging economy. In the case of the positive steady states, the relative level of Country 2 to Country 1 is decreasing in the large scale sector's market power, and increasing in its innovation productivity, while the relative level of the small scale sector remains constant in the case considered here. Increases in the small sector innovation productivity $\tilde{\mu}_{2S}$, representing for example the effect of policies addressing the public good nature of absorption, always benefit the small scale sector. and mostly also the large scale sector. At positive steady states of type A, they raise both the relative level a_2^* of the small scale sector to its large scale sectors, and the relative level b^* of the domestic large scale sector relative to the leading large scale sector. At the zero cases the relative level a_2^{**} of the small scale sector is also increasing $\tilde{\mu}_{2S}$, but is decreasing in large scale sector innovation productivity $\tilde{\mu}_{2L}$ and χ_2. Hence also the large and small scale sector growth rates are increasing in $\tilde{\mu}_{2S}$ if $a_2^{**}>0$. Only when both steady states are zero are the growth rates independent, each depending solely on its own innovation productivity $\tilde{\mu}_{2S}, \tilde{\mu}_{2L}$.

Summarizing, the results show that wages depend on both innovation and absorption. Each of these technological processes requires for optimality differently motivated public policies in physical and human capital, technology, infrastructure, and so on.

Theorem 5 shows, without any additional assumptions, that in underdeveloped countries the small scale sector tends to be more backward relative to the large scale sector than in developed countries, because it has better access to spillovers from developed R&D. This contributes to an explanation for the large informal sectors in underdeveloped countries.

Theorem 6 compares the private rates of innovation investment with those that would be optimal for a benevolent government. The government maximizes the rate of change of aggregate income, consistently with our perfect myopic foresight framework. The presence of market power makes the innovation investment-to-income ratio inefficient, since the impact of cost savings is understated. As market power decreases, innovation tends to efficiency. By contrast, the innovation investment-to-income ratio is efficient for absorption. However, this does not take into account 1) the fact that the effort expended in technology absorption is repeated in all competing firms, 2) that government investment in skills useful to the small scale sector can increase the absorption rate. That the aggregate income level is reduced by market power implies an additional inefficiency factor in both innovation and absorption.

Continuing with Theorem 6, in our simplified context a tax on market power improves the efficiency of innovation. In a more general context, a market power tax can allow a government to select a mark up consistent with an optimal level of market power.

Alternatively, given fixed or endogenous levels of market power, the government can reach optimum innovation (but not production) with a balanced budget by using an appropriate combination of taxes on profits and subsidies on innovation, requiring in addition a lump sum tax on wages for low profit levels. This policy for optimizing innovation can complement a market power tax, that can only approximate efficiency.

The results and discussion show that free market policies are suboptimal for developed and underdeveloped industrial market economies, in levels, growth rates, wages, and equity. These can all be improved by taxing profits and subsidizing innovation and absorption, consistently with active public science and human capital policies.

The model shows that optimal technology policies need to be two-pronged, supporting both innovation and absorption. Supporting absorption will benefit both sectors, while supporting innovation tends to increase growth without reducing inequality, and might for cases with low knowledge externalities, if the small sector is relatively large and diverging in growth rates, even decrease the small scale sector growth rate, when the disadvantage of backwardness becomes relatively larger.

The model can serve to understand a series of issues of developed and underdeveloped industrial market economies, under the contradictory impacts of innovation and competition on welfare and distribution, such as pro-poor growth, global income concentration, increased corporate political influence under deregulation, sustainability in the face of both poverty and corporate power, the global economic business cycle, informality, and so on.

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