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Corrigendum to “Lyapunov functions for a class of nonlinear systems using caputo derivative” [Commun Nonlinear Sci Numer Simulat 43 (2017) 91–99]

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In this text we report and correct some mistakes made in our paper [1]. Lemma 3 in [1] is wrong (and consequently Remark 2 and Corollary 1 are also incorret) since we used a result introduced in [2] which is not applicable in the analysis we did. Particularly, we claimed that $\int_{t_0}^{t} \frac{y(\tau)y'(\tau)}{(t-\tau)^\alpha} d\tau \le 0$ where $y(\tau) = (n-1)x^{n-1}(t) - x(\tau)$ and $\int_{t_0}^{t} \frac{z(\tau)z'(\tau)}{(t-\tau)^\alpha} d\tau \le 0$ where $z(\tau) = mx^m(t) - x^m(\tau)$, but these statements are not necessarily true. Considering these errors, the corrected version of Lemma 3 in [1] is presented next.

Lemma 1. *Let $x(t) : \mathbb{R} \to \mathbb{R}$ be an absolutely continuous and differentiable function. Then, $\forall \alpha \in (0, 1)$, the following relationships hold:*

(i) *If $x(t)$ is a monotonically increasing or monotonically decreasing function then $\frac{n-1}{n}\, {}_{t_0}^{C}D_t^{\alpha} x^n(t) \le x(t)\, {}_{t_0}^{C}D_t^{\alpha} x^{n-1}(t)$, where $n \in \{2k, k \in \mathbb{N} - \{0, 1\}\}$.*
(ii) *$\frac{1}{2}\, {}_{t_0}^{C}D_t^{\alpha} x^{2m}(t) \le x^m(t)\, {}_{t_0}^{C}D_t^{\alpha} x^m(t)$, where $m \in \{N - \{0\}\}$.*

Proof.

(i) Let $A(t) = \frac{1}{n}\, {}_{t_0}^{C}D_t^{\alpha} x^n(t) - \frac{x(t)}{n-1}\, {}_{t_0}^{C}D_t^{\alpha} x^{n-1}(t)$. Using the definition of the Caputo derivative we can write

$$A(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{[x(\tau) - x(t)]x^{n-2}(\tau)x'(\tau)}{(t-\tau)^\alpha} d\tau. \tag{1}$$

Introducing the change of variables: $y(\tau) = x(\tau) - x(t)$, $\frac{dy(\tau)}{d\tau} = \frac{dx(\tau)}{d\tau}$, we can rewrite (1) as

$$A(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{y(\tau)x^{n-2}(\tau)y'(\tau)}{(t-\tau)^\alpha} d\tau. \tag{2}$$

Integrating (2) by parts we obtain

$$A(t) = x^{n-2}(\tau)F(\tau)\Big|_{t_0}^{t} - (n-2)\int_{t_0}^{t} x^{n-3}(\tau)F(\tau)d\tau, \tag{3}$$

where

$$F(\tau) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{\tau} \frac{y(\tau)y'(\tau)}{(t-\tau)^\alpha} d\tau = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{\tau} \frac{[x(\tau) - x(t)]x'(\tau)}{(t-\tau)^\alpha} d\tau.$$

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Note that $t_0 \leq \tau \leq t$. It is clear that if $x(t)$ is monotonically increasing or monotonically decreasing then $x(\tau) - x(t)$ and $x'(\tau)$ will have different signs; hence $F(\tau) \leq 0$. From this point, the proof is exactly the same as the original proof of Lemma 3 i) in [1], starting from equation (10) of that article.

(ii) This proof is very similar to the one done for Lemma 3 ii) in [1], with the difference that now the appropriate change of variables is $y(\tau) = x^m(t) - x^m(\tau)$. Thus, the integral $\int_{t_0}^{t} \frac{y(\tau)y'(\tau)}{(t-\tau)^\alpha} d\tau$ is in fact nonpositive, which can be shown using the arguments of Lemma 1 in [2].

□

Remark 1. If ${}_{t_0}^{C}D_t^\alpha x^{2m}(t) \geq 0$ it is obvious form Lemma 1 ii) that $\frac{1}{2m}\, {}_{t_0}^{C}D_t^\alpha x^{2m}(t) \leq x^m(t)\, {}_{t_0}^{C}D_t^\alpha x^m(t)$, which recovers Lemma 3 ii) of [1].

With the changes indicated in the above lines, the corrected versions of Remark 2 and Corollary 1 in [1] are presented next.

Remark 2. If $x(t) \geq 0$ and monotonically increasing or decreasing, then Lemma 1 i) is valid for odd n. Possibly using other hypotheses, we believe that this result can be generalized for different functions (not necessarily monotonically increasing or decreasing) and for $n \in \mathbb{R},\ n \geq 1$.

Corollary 1. *Let $x(t) : \mathbb{R} \to \mathbb{R}$ be an absolutely continuous and differentiable function. Then, $\forall \alpha \in (0, 1)$, the following relationships hold:*

(i) *If $x(t)$ is a monotonically increasing or monotonically decreasing function and $x(t) \geq 0$ then $\frac{1}{n}\, {}_{t_0}^{C}D_t^\alpha x^n(t) \leq x^{n-1}(t)\, {}_{t_0}^{C}D_t^\alpha x(t)$, where $n \in \{N - \{0\}\}$.*
(ii) ${}_{t_0}^{C}D_t^\alpha x^{2^m}(t) \leq 2^m x^{(2^m-1)}(t)\, {}_{t_0}^{C}D_t^\alpha x(t)$, *where $m \in \{N - \{0\}\}$.*

Proof. The proofs of both statements consist in iterating $n - 2$ times the inequalities of Lemma 1, as it was done in the proof of Corollary 1 in [1]. □

Despite the corrections mentioned for various results presented in [1], Proposition 1 of that article remains valid, adding to part i) the hypothesis that $x(t)$ is monotonically increasing or decreasing. Also, all the Examples presented in [1] are still useful to show the applicability of the corrected results, especially of Corollary 1 i), now using different Lyapunov functions; for Examples 1, 2 we could use $V(t) = x_1^2(t) + \frac{6!}{16} x_2^{16}(t)$ and for Example 3 the function $V(t) = x_1^{46}(t) + x_2^{46}(t)$ would work.

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References

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