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Preservation of Formation Tracking under Incomplete Information and Chaotic Path-following ⋆

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Abstract: This paper presents a formation tracking strategy based on local potential functions and the leader-followers scheme for a group of point robots moving in the space. A leader robot is chosen to follow a prescribed trajectory whilst the rest, considered as followers, are formed in an open chain configuration. The formation convergence and the path following, including chaotic trajectories, are guaranteed if every follower robot measures exactly the velocity of the next robot and the leader knows the velocity of the marching path. Then, we analyze the preservation of the formation using approximations of the velocities of robots and the chaotic path. These restrictions appear in real implementations in robots equipped by position sensors only and where the velocities functions are approximated by online numerical methods.

Keywords: Mobile robots, Chaos, Formation Control, Path-following, Multi-robot systems.

1. INTRODUCTION

Formation control is related to the movement coordination strategies for groups of mobile robots to achieve geometric patterns within a workspace (Chen and Wang, 2005). The control laws are decentralized because every robot possesses a local control device which can sense only the position of certain team members. The decentralized schemes permit more autonomy for the robots, less computational load in control implementations and its applicability to large scale groups (Do, 2007). Applications such as transportation and manipulation of large objects, searching and rescue tasks, perimeter detection, etc, require the tracking of some trajectories of the group, preserving the formation pattern simultaneously. This problem is referred as formation tracking (Do, 2007), formation path following (Ghommam et al., 2010), marching control (Hernandez-Martinez and Aranda-Bricaire, 2009) or flocking behavior (Regmi et al., 2005). Some approaches include behavior-based, navigation functions, virtual structure and the leader-followers strategies (Kostic et al., 2010). The control strategies differ with respect to the information communicated to the robots about the trajectory (marching path) or the velocities of robots that guarantee the formation preserving during the path-following, conserving the greatest possible decentralization.

The leader-followers schemes are frequently studied by the academic community due the biological behavior inspiration and the military applications. A leader robot is assigned to follow the trajectory whilst the rest, termed followers, converge to some formation pattern with respect to the leader. For instance, Belkhouche and Belkhouche (2005) studies the case of a convoy-like formation, where the robots are placed at the desired positions initially, and they know the linear and angular velocities of the next robot. Predictive Control is applied in Weihua and Go (2010) to add a term to the cost function of the leader to preserve the formation. Consolini et al. (2008), define a control law where the positions of followers during the path following are not rigidly fixed with respect to the leader, but vary in a ball centered in the leader reference frame. Some works are based on the existence of a virtual leader, where the reference of the robots is based on the complete information of the position and orientation of some virtual entities (Porfiri et al., 2007). To estimate the velocity of the leader, in Do (2009) is proposed a reduced order observer and Peng et al. (2011) uses its online approximation through local sensors with line-of-sight range. Finally, Hernandez-Martinez and Aranda-Bricaire (2009) and Ghommam et al. (2010) add the path derivative to all robots to preserve the formation during the path following.

In this paper, we analyze a formation tracking of n point modeled as point robots or omnidirectional robots in the space under the leader-followers scheme, with the next assumptions:

(1) The first $n-1$ robots are formed with respect to the leader (robot R_n) in an open chain or convoy configuration, i.e. the robot R_i can measure only the

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position, with respect to a global reference frame, of the robot R_{i+1}.

(2) The leader robot R_n is assigned to follow the marching trajectory, which can be a sufficiently smooth function.

(3) The velocities of the robots and marching trajectory are estimated numerically by the local controllers of the robots.

The previous conditions perform a decentralized scheme where different tasks are assigned for the leader and followers sharing the minimum information. Note that the trajectory generator of the leader robot can produce complex behavior. It can be emulated by a chaotic function of time. Thus, the analysis of the group behavior under chaotic path-following is based in the next paradigm: if the formation is preserved in the presence of chaotic systems, then the control objective is achieved using another trajectories. Few works include chaos in formation control, for example, Wenwu et al. (2010) studie the presence of chaos due to the appearance of time delay, and Ji and Wei (2011) analyze the chaos in the synchronization of nonlinear networked systems with switching topology. Also, Shi et al. (2006) combine a chaotic optimization algorithm with the artificial potential field method to generate local optimal path. This paper continues the previous work of Hernandez-Martinez and Aranda-Bricaire (2010), adding the case of point robots following a chaotic trajectory in $3D$ using numerical approximations of the velocities of the robots and trajectory. These restrictions close the control strategy to a more realistic control implementations considering the effects of incomplete information.

The rest of the paper is organized as follows. Section 2 introduces the kinematic model and the problem statement. The formation tracking control strategy with exact information about velocities is presented in Section 3. The control strategy based on the approximation of velocities is analyzed in 4. Finally, some concluding remarks are presented in Section 5.

2. PROBLEM DEFINITION

Denote by $N = \{R_1, ..., R_n\}$, a set of n point robots moving in the space with positions $z_i(t) = [z_{1i}(t), z_{2i}(t), z_{3i}(t)]^T$, $i = 1, ..., n$. The kinematic model of each agent or robot R_i is described by

$$\dot{z}_i = u_i, i = 1, ..., n, \tag{1}$$

where $u_i \in \Re^3$ is the velocity along the X, Y and Z axis of the i-th robot. Consider to $R_1, ..., R_{n-1}$ the follower robots, and R_n the leader robot. Based on the leader-follower scheme, define z_i^* the desired relative position of R_i in a particular formation. In this work, we establish the z_i^* as

$$z_i^* = z_{i+1} + c_{(i+1)i}, i = 1, ..., n-1, \qquad z_n^* = m(t), \tag{2}$$

where $c_{(i+1)i} \in \Re^3$, $i = 1, ..., n-1$ denotes the vector which represents the desired relative position of R_i with respect to R_{i+1} and $m(t) = [m_X(t), m_Y(t), m_Z(t)] \in \Re^3$ is the marching path (twice differentiable function) of the leader. Figure 1 shows and example where the robots satisfy a specific formation pattern and the path-following. Note that the goal of the leader is to follow the path of marching whereas the goal of the followers is to maintain a desired pattern formation with respect to the leader using the position of another robot. This strategy is usually considered as an open-chain or convoy configuration (Belkhouche and Belkhouche, 2005).

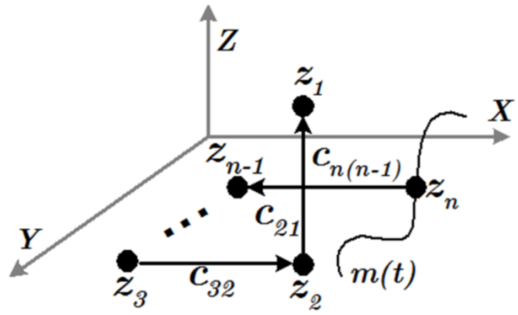


Fig. 1. Desired formation tracking of the robots.

Problem Statement. The goal of formation tracking is to design a control law $u_i = f_i(z_i, z_i^*)$ for every robot R_i, such that $\lim_{t\to\infty}(z_i - z_i^*) = 0$, $i = 1, ..., (n-1)$ and $\lim_{t\to\infty}(z_n - z_n^*) = 0$.

3. CONTROL STRATEGY BASED ON THE VELOCITY OF THE NEXT ROBOT

Based on Hernandez-Martinez and Aranda-Bricaire (2010), for every follower robot, Local Potential Functions LPF's are established by

$$\gamma_i = \|z_i - z_i^*\|^2, i = 1, ..., n-1 \tag{3}$$

Note that γ_i is always positive and reaches its minimum only when $z_i = z_i^*$. The approach of (attractive) LPF consists of applying the partial derivative of a LPF with respect to z_i, as control inputs of every robot R_i. Thus, the control inputs steer every robot to the minimum of this potential function which is designed according to the specific position vectors that construct a particular formation. So, using these functions, a control strategy is defined by

$$u_i = -\frac{1}{2}k\frac{\partial \gamma_i}{\partial z_i} + \dot{z}_{i+1}, i = 1, ..., n-1 \qquad u_n = \dot{m}(t) - k_m(z_n - m(t)) \tag{4}$$

The control scheme is illustrated in Fig. 2. Note that the control law of the leader needs the exact derivative of $m(t)$ whilst the followers require the knowledge of $\dot{z}_{i+1}$.

Proposition 1. Consider the system (1) and the control law (4) for n robots. Suppose that $k, k_m > 0$. Then, in the closed-loop system (1), (4), the first $n-1$ robots converge to the desired formation i.e. $\lim_{t\to\infty}(z_i - z_i^*) = 0$, $i = 1, ..., n-1$, whereas R_n converges to the path of marching i.e. $\lim_{t\to\infty}(z_n(t) - m(t)) = 0$.

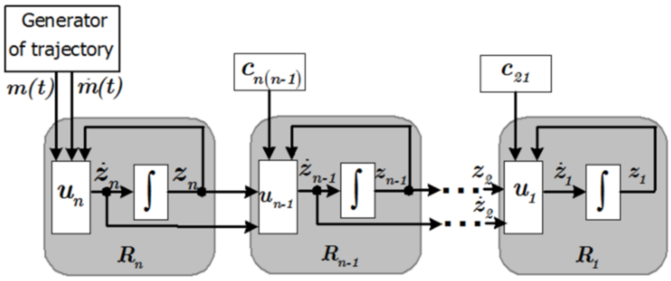


Fig. 2. Formation tracking scheme with the exact information about velocities of robots and trajectory.

Proof. The dynamics of z_i in the closed-loop system (1)-(4) is given by

$$\dot{z}_i = -k\left(z_i - z_{i+1} - c_{(i+1)i}\right) + \dot{z}_{i+1}, i = 1, ..., n-1 \quad (5)$$
$$\dot{z}_n = \dot{m}(t) - k_m(\alpha_n - m(t)) \quad (6)$$

Define the error coordinates by

$$e_i = z_i - z_i^*, \quad i = 1, ..., n \quad (7)$$

where $e_1, ..., e_{n-1}$ are the formation errors of the followers and e_n is named the path-following error of the leader. The dynamics of the error coordinates (7) is given by

$$\dot{e} = (B \otimes I_3)\, e \quad (8)$$

where $\otimes$ denotes the Kronecker product (the Kronecker product allows a more compact notation for systems' equations), I_3 is the 3×3 identity matrix and $B = diag[-k, -k, ..., -k_m]$. Clearly, the matrix B is Hurwitz and the errors converges to zero. This means that the $n-1$ first agents converges to the desired formation whereas R_n converges to the marching path. ■

Remark 1. It is necessary to add $\dot{z}_{i+1}$ to the followers to guarantee the formation is preserved during the trajectory tracking. If they are not added, the dynamics of the errors becomes $\dot{e} = (B \otimes I_3)\, e + [S_x, S_y, S_z]^T$, with $S_X = [0, ..., 0, \dot{m_X}(t), 0]^T$, $S_Y = [0, ..., 0, \dot{m_Y}(t), 0]^T$ and $S_Z = [0, ..., 0, \dot{m_Z}(t), 0]^T$. Even though the matrix B is clearly Hurwitz, the dynamics of R_{n-1} is disturbed by the velocity of marching and it is transmitted to the other followers. This is an example of disturbances in chain-stability in formation control using the leader-followers scheme (Tanner et al., 2004), where the motion of the leader influences the formation stability of the followers.

The first $n-1$ robots do not require to process complete information about $m(t)$ and the positions of all robots, different to Yamaguchi (2003); Do (2007), where all robots must know the target position or trajectory and more than one desired distance between robots. In Hernandez-Martinez and Aranda-Bricaire (2009), a similar control law is proposed adding the velocity of the marching path instead the velocity of the next robot. This strategy also guarantees the convergence of the formation errors.

3.1 Numerical simulation

The control law strategy (4) is appropriated for any kind of sufficiently smooth function, including chaotic functions. Figure 3 shows a numerical simulation for the closed-loop system (1)-(4) for $n = 3$, $k = 0.5$ and $k_m = 1$. The initial conditions are given by $z_1(0) = [2, 4, -6]$, $z_2(0) = [4, 4, -6]$ and $z_3(0) = [6, 4, -6]$. The desired formation is a line (parallel to Z axis) defined by the vectors $c_{21} = c_{32} = [0, 0, -1]$ and the marching path is a chaotic Lorenz system, reported in Sangpet and Kuntanapreed (2010), defined by the solution of the dynamical system

$$\begin{aligned} \dot{m}_X &= 10(m_Y - m_X) \\ \dot{m}_Y &= 28m_X - m_Y - m_X m_Z \\ \dot{m}_Z &= m_X m_Y - \frac{8}{3} m_Z \end{aligned} \quad (9)$$

where $m_X(0) = 1$, $m_Y(0) = 2$ and $m_Z(0) = 3$. In the fig. 3(a) the trajectories of the robots converge to the desired formation pattern and the desired marching path. It is shown in fig. 3(b) through the convergence of the error coordinates. Note that the convergence of the errors is reached due to the strong assumption that the velocities of the robots and the marching path is measured exactly. It becomes in more communication items for the local controllers of robots. Next section analyzes the error convergence in the case of approximations of the marching path and robots velocities.

4. CONTROL STRATEGY USING APPROXIMATIONS OF VELOCITIES

The control scheme now is modified according to the fig. 4. Note that leader and follower unknown the exact velocities of the marching path or the next robot, respectively. Then, the velocities are calculated by the local controllers using numerical methods, performing a more realistic and decentralized scheme. The derivative approximation function is defined, in the frequency domain, as

$$g(s) = \frac{s}{\tau s + 1} \quad (10)$$

where $\tau \in \Re$ and $\tau \geq 0$. Thus, the approximation of the next robot velocity (for the followers) and the marching path velocity (for the leader) is established, respectively, by

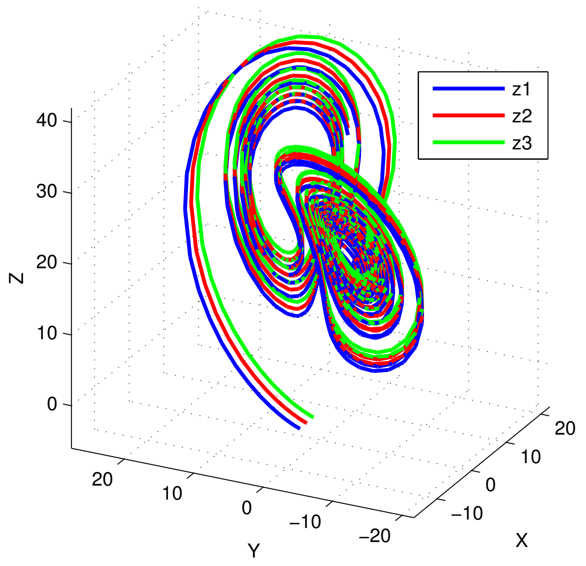
$$\begin{aligned} \varphi_i\,(s) &= g(s) z_i\,(s)\,, i = 2, ..., n \\ \varphi_m\,(s) &= g(s) m\,(s) \end{aligned} \quad (11)$$

and expressed in state-space variables, they become

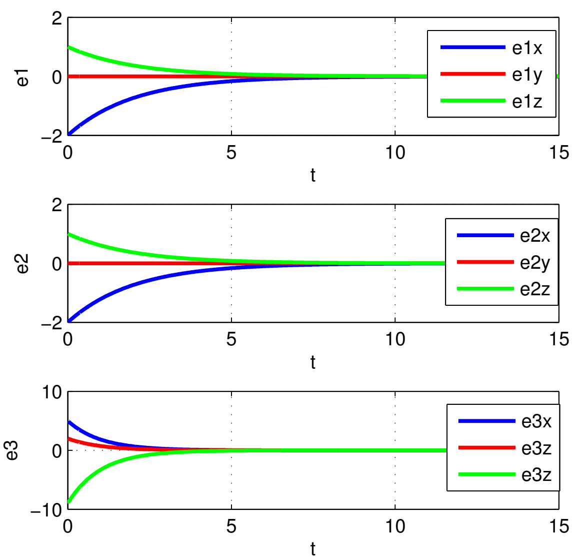
$$\begin{aligned} \varphi_i &= -\frac{1}{\tau}\eta_i + \frac{1}{\tau} z_i, i = 2, ..., n \\ \dot{\eta}_i &= -\frac{1}{\tau}\eta_i + \frac{1}{\tau} z_i, i = 2, ..., n \\ \varphi_m &= -\frac{1}{\tau}\eta_m + \frac{1}{\tau} m\,(t) \\ \dot{\eta}_m &= -\frac{1}{\tau}\eta_m + \frac{1}{\tau} m\,(t) \end{aligned} \quad (12)$$

Then, the control law (4) is modified to

$$\begin{aligned} u_i &= -\frac{1}{2} k \frac{\partial \gamma_i}{\partial z_i} + \varphi_{i+1}, i = 1, ..., n-1 \\ u_n &= -k_m(z_n - m(t)) + \varphi_m \end{aligned} \quad (13)$$



(a) Trajectories of robots



(b) Graphics of error coordinates

Fig. 3. Simulation of chaotic marching path

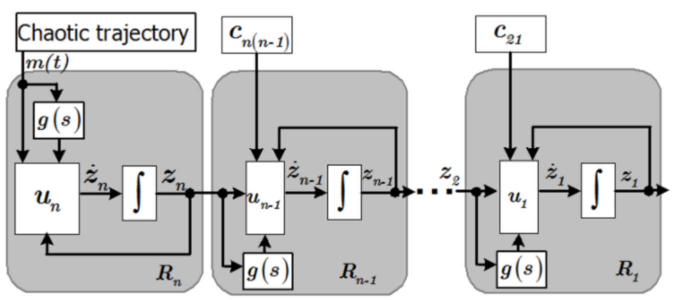


Fig. 4. Formation tracking scheme using approximation of velocities of robots and marching trajectory.

The closed-loop system (1)-(13), results in the extended system's equations

$$\begin{aligned}
\dot{z}_i &= -k\left(z_i - z_{i+1} - c_{(i+1)i}\right) - \frac{1}{\tau}\eta_{i+1} + \frac{1}{\tau}z_{i+1}, \quad (14)\\
i &= 1, ..., n-1\\
\dot{z}_n &= -k_m\left(z_n - m\left(t\right)\right) - \frac{1}{\tau}\eta_m + \frac{1}{\tau}m\left(t\right)\\
\dot{\eta}_i &= -\frac{1}{\tau}\eta_i + \frac{1}{\tau}z_i, i = 2, ..., n\\
\dot{\eta}_m &= -\frac{1}{\tau}\eta_m + \frac{1}{\tau}m\left(t\right)
\end{aligned}$$

and written in matricial form

$$\begin{bmatrix}\dot{z}\\ \dot{\eta}\end{bmatrix} = (A \otimes I_3)\begin{bmatrix}z\\ \eta\end{bmatrix} + \begin{bmatrix}C\\ M\end{bmatrix} \quad (15)$$

where $z = [z_1, ..., z_n]^T$, $\eta = [\eta_2, ..., \eta_n, \eta_m]^T$, $M = [0, ..., 0, \frac{1}{\tau}m(t)]^T$, $A = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}$ with $A_{12} = A_{22} = diag\left[-\frac{1}{\tau}, ..., -\frac{1}{\tau}\right]$,

$$A_{11} = \begin{bmatrix}-k & \left(k+\frac{1}{\tau}\right) & 0 & 0 & \cdots & 0\\ 0 & -k & \left(k+\frac{1}{\tau}\right) & 0 & \cdots & 0\\ \vdots & & & \ddots & & \vdots\\ 0 & 0 & 0 & \cdots & -k & \left(k+\frac{1}{\tau}\right)\\ 0 & 0 & 0 & \cdots & 0 & -k_m\end{bmatrix},$$

$$A_{21} = \begin{bmatrix}0 & \frac{1}{\tau} & 0 & 0 & \cdots & 0\\ 0 & 0 & \frac{1}{\tau} & 0 & \cdots & 0\\ \vdots & & & \ddots & & \vdots\\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{\tau}\\ 0 & 0 & 0 & \cdots & 0 & 0\end{bmatrix}, C = \begin{bmatrix}kc_{21}\\ \vdots\\ kc_{n(n-1)}\\ \left(k_m + \frac{1}{\tau}\right)m\left(t\right)\end{bmatrix}.$$

It is clear that the inaccurate information about the velocities, does not ensure exactly the convergence of the formation and path-following errors. However, the next result establishes a proposition about the boundedness of the errors around zero based on small value of τ and the behavior of the acceleration of the marching path.

Proposition 2. Consider the system (1) and the control law (13) for n robots. Suppose that $k, k_m > 0$. Then, in the closed-loop system (1), (13), the formation errors and the path-following error of the leader are bounded for any $\tau \geq 0$ and $|\ddot{m}(t)| < \infty$. Furthermore, if $\ddot{m}(t) = 0$, the errors tend exponentially to zero regardless of the value of τ and when $|\ddot{m}(t)| < \infty$ and $\tau \to 0$, the errors converge to zero as $t \to \infty$ also.

Proof. Defining the error coordinates

$$
\begin{aligned}
e_i &= z_i - z_i^*, i = 1, ..., n \\
\delta_i &= \varphi_i - \varphi_{i+1}, i = 2, ..., n-1 \\
\delta_n &= \varphi_n - \varphi_m(t) \\
\delta_{n+1} &= \varphi_m - \dot{m}(t)
\end{aligned}
$$

Note that the errors e_i are similar to (7) and the δ_i errors are the difference between the approximated velocities of the robots and the marching path. Thus, if the e_i converge to zero, the robots achieve the desired formation and the convergence of δ_i ensures that the robots are moved to the same velocity of the marching path, i.e. preserving the formation pattern. The dynamics of the error coordinates is given by

$$
\begin{aligned}
\dot{e}_i &= -ke_i + ke_{i+1} + \delta_{i+1}, i = 1, ..., n-2 \qquad (16) \\
\dot{e}_{n-1} &= -ke_{n-1} + k_m e_n + \delta_n \\
\dot{e}_n &= -k_m e_n + \delta_{n+1} \\
\dot{\delta}_i &= -\frac{1}{\tau}\left(ke_i - ke_{i+1} + \delta_i - \delta_{i+1}\right), i = 2, ..., n-2 \\
\dot{\delta}_{n-1} &= -\frac{1}{\tau}\left(ke_{n-1} - k_m e_n + \delta_{n-1} - \delta_n\right) \\
\dot{\delta}_n &= -\frac{1}{\tau}\left(k_m e_n + \delta_n - \delta_{n+1}\right) \\
\dot{\delta}_{n+1} &= -\frac{1}{\tau}\delta_{n+1} - \dddot{m}(t)
\end{aligned}
$$

and written in matricial form

$$
\begin{bmatrix} \dot{e} \\ \dot{\delta} \end{bmatrix} = (\tilde{B} \otimes I_3) \begin{bmatrix} e \\ \delta \end{bmatrix} - \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} \dddot{m}(t) \qquad (17)
$$

where $e = [e_1, ..., e_n]^T$, $\delta = [\delta_2, ..., \delta_n, \delta_{n+1}]^T$, $\tilde{B} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$ with B_{12} is the identity matrix of $n \times n$,

$$
B_{11} = \begin{bmatrix} -k & k & 0 & 0 & \cdots & 0 \\ 0 & -k & k & 0 & \cdots & 0 \\ & & \ddots & & & \vdots \\ 0 & 0 & 0 & \cdots & -k & k_m \\ 0 & 0 & 0 & \cdots & 0 & -k_m \end{bmatrix},
$$

$$
B_{21} = \frac{1}{\tau} \begin{bmatrix} 0 & -k & k & 0 & \cdots & 0 \\ 0 & 0 & -k & k & \cdots & 0 \\ & & \ddots & & & \vdots \\ 0 & 0 & 0 & \cdots & -k & k_m \\ 0 & 0 & 0 & \cdots & 0 & -k_m \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix},
$$

$$
B_{22} = \frac{1}{\tau} \begin{bmatrix} -1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 \\ & & \ddots & & & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & -1 \end{bmatrix}.
$$

By using mathematical induction, it can be shown that the characteristic polynomial of matrix $\tilde{B}$ is given by $p_{\tilde{B}}(\lambda) = (\lambda + 1)^{3n}(\lambda + k)^{3(n-1)}(\lambda + k_m)^3$, therefore we conclude that (17) is bounded-input bounded-state stable. Observe that eigenvalues do not depend on the value of τ. Note that if $\dddot{m}(t) = 0$, we have an homogeneous linear system. Therefore, the solution converge exponentially to zero. Finally, note that the solution, as $t \to \infty$, of the coordinate δ_{n+1} is bounded by

$$
|\delta_{n+1}| \leq \tau \, |\dddot{m}(t)| \qquad (18)
$$

from which we can see that $|\delta_{n+1}| \to 0$ as $\tau \to 0$. ■

4.1 Numerical simulation

Figure 5 shows a numerical simulation for the closed-loop system (1)-(13) for $n = 3$, $k = 0.5$, $k_m = 1$, $\tau = 0.001$ and the same initial conditions and chaotic marching path than the previous simulation. The errors coordinates do not converge to zero, but remains bounded around zero.

Define the error performance index of the coordinates e_i and δ_i as $\tilde{e} = \frac{1}{t_f}\int_0^{t_f}\left(\sqrt{\|e_1\|^2 + \|e_2\|^2 + \|e_3\|^2}\right)dt$ and $\tilde{\delta} = \frac{1}{t_f}\int_0^{t_f}\left(\sqrt{\|\delta_2\|^2 + \|\delta_3\|^2 + \|\delta_4\|^2}\right)dt$, respectively. Table 1 shows the results with different values of τ for the same Lorenz system with $t_f = 15$. We observe the best results for smaller values of τ.

Table 1. Error performance index with different values of τ

τ	$\tilde{e}$	$\tilde{\delta}$
0.01	2.52	16.976
0.001	1.58	3.141
0.0001	1.52	1.929

5. CONCLUSION

This paper shows that under the assumption of perfect knowledge about the positions and velocities of the robots and the marching path, the control strategy guarantees the convergence of the errors, including chaotic behavior of the trajectory. Also we have shown that, when the velocities are approximated, the errors are bounded if the trajectory is sufficiently smooth; the bound of the errors improves when the bandwidth approximation increases for the velocities recovering the ideal case in the limit.

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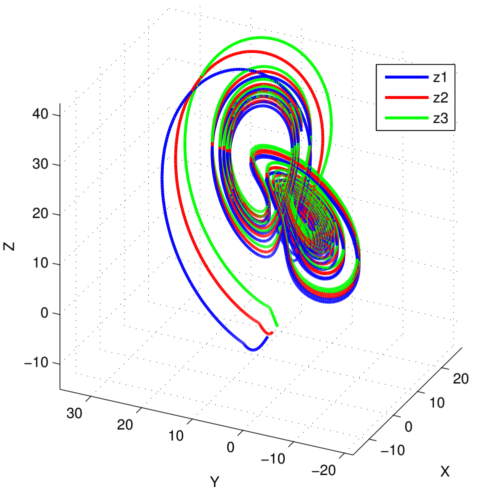
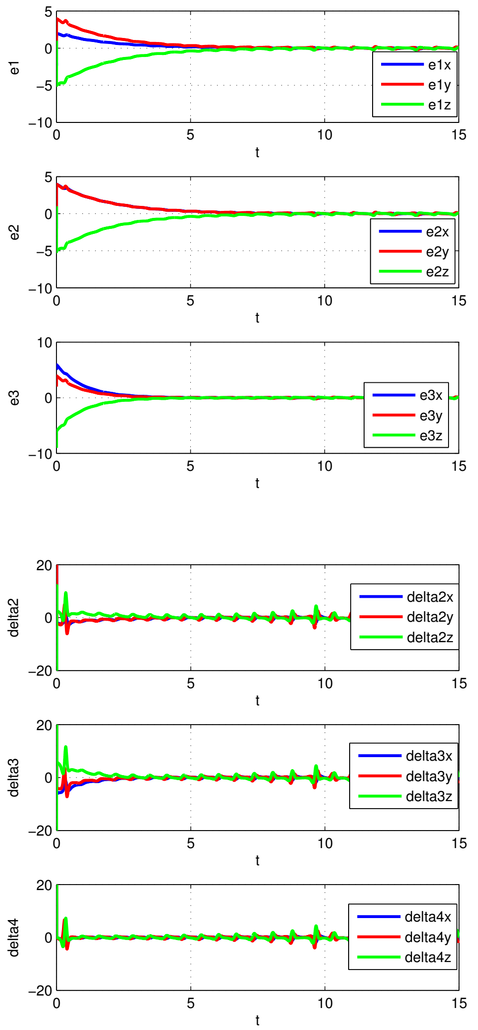


(a) Trajectories of robots



(b) Graphics of error coordinates

Fig. 5. Simulation of marching control with approximations of the velocities

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