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Formation Control of Multiple Robots Avoiding Local Minima ⋆

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Abstract: Recently, distance based formation of groups of mobile robots is attracting researchers to provide a better format for decentralized control strategies. However, the setup formulation introduces rigidity problems and additional undesired local minima in the formation convergence standard algorithms. This paper explores the use of combined distance-based attractive-repulsive potentials to simplify the navigation problem as well as the use of angular information between robots to avoid unwanted formation patterns that verify the distance constraints. The proposed algorithms are analyzed for the case of omnidirectional robots in a two-dimensional environment and tested by numerical simulations showing a sliding mode behavior to reach the intended formation configuration.

Keywords: Mobile robots; Formation control; Artificial Potential Function; Collision Avoidance; Lyapunov

1. INTRODUCTION

The study of behaviors between groups of mobile agents has been the subject of investigation since many years now. It has a scope and complexity that generates interesting problems from an academic point of view but also helps in the solution of real problems in many areas, from navigation systems to safety and security applications [Cao et al., 1997].

Behaviors, seeing as the interactions between robots and their environment, include not only motion coordination but also material handling, task decomposition and resource allocation, etc. [Arkin, 1998].

In the case of motion coordination for a group of robots the desired pattern or space distribution of the robots in their environment is usually called a *formation*. Formation control is referred to as finding a motion strategy that leads to convergence to a desired formation whereas the group trajectory is addressed as formation tracking, flocking behavior or marching control [Ren and Beard, 2008].

Behavior based control uses biological inspired mechanisms to design motion laws that achieve the desired goals [Balch and Arkin, 1998]. Other strategies are based on the use of graph theory to define a formation and study its main characteristics. Using this approach it is possible to specify which robots interact with each other through a communication graph. Global convergence can be analyzed using the properties of the Laplacian matrix of the communication graph between robots in the formation [Desai, 2002]. Decentralized algorithms have been designed to solve the problem of partial information at the robot level [Muhammad and Egerstedt, 2004].

Another powerful technique to design control laws comes from the use of artificial vector fields where formation control and collision avoidance can be integrated in one framework and convergence shown by the use of Lyapunov theory [Dimarogonas and Kyriakopoulos, 2006], [Hernández-Martínez and Aranda-Bricaire, 2011]. Feedback Linearization techniques have been used when robot kinematics are more complex [Liu and Jiang, 2013].

The standard formation defines the positions between robots as vectors. Distance based formation control deals with formations where only the scalar distance between robots is specified. The latter are more flexible but, that feature may represent an advantage or disadvantage depending on the application domain. In both cases rigidity problems arise [Krick et al., 2009]. Convergence of distance based formations has been reported in many works recently. For example, convergence is shown in [Dimarogonas and Johansson, 2008] for the case where the formation graph is a tree and therefore non rigid. Graphs associated to rigid formations have cycles in them which introduce convergence problems in the formation as can

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be seen in [Dimarogonas and Johansson, 2009]. Adaptive control has also been used in the case of robot parametric uncertainties [Cai and de Queiroz, 2014].

In this work a modified distance based formation control problem is presented that restrict the formation configuration domain using angular information between robots. In this way a new control strategy, based on artificial potential fields is developed improving convergence characteristics over the standard algorithms. Simulations for the case of three omnidirectional robots show the feasibility of the approach. The paper is organized as follows. Section 2 formulates the modified problem to be solved and defines the potential fields used. In Section 3, the formation control laws are established for the cases under consideration. Section 4 includes simulation results of the control laws designed. Finally, section 5 presents some conclusion remarks and future work.

2. PROBLEM DEFINITION

2.1 Distance-based Formation Problem

Let $\mathbf{q} = [\mathbf{q_1^t}, \ldots, \mathbf{q_N^t}]^t \in \mathbb{R}^{2N}$ be the position of the center of N two-dimensional omnidirectional robots of diameter ρ_i with the simple integrator dynamics given by the following equation:

$$\dot{\mathbf{q}}_\mathbf{i} = \mathbf{u_i},\ i = 1, \ldots, N \tag{1}$$

Let $\mathbf{r_{ij}} = \mathbf{q_j} - \mathbf{q_i}$ be the relative position vector of robot j with respect to robot i and $r_{ij} = \|\mathbf{r_{ij}}\|$ be the Euclidean distance between robots i and j.

The distance-based formation problem can be stated as:

Definition 1. Given N omnidirectional robots defined by (1), find a control law for $\mathbf{u_i}, \forall i$ such that $r_{ij}(t)$ tends to prespecified desired distances d_{ij}.

Not all distances may be prespecified leading to more flexible configurations, invariant against rotations and symmetries on the plane. Even non rigid configurations are possible if not enough distances d_{ij} are set.

Let $N_i, i = 1, \ldots, N$ be the set of robot indexes for which d_{ij} is given and let Φ be the set of all robot configurations satisfying the distance constraints. It is assumed that $d_{ij} = d_{ji}$ are both specified. Furthermore, for each configuration $\phi_n \in \Phi$ there is a set

$$\Gamma_n = \left\{ \alpha^n_{ijk} = \widehat{\mathbf{r^n_{ij}} \cdot \mathbf{r^n_{ik}}}, \forall i,j,k \mid j,k \in N_i \right\} \tag{2}$$

where α^n_{ijk} are the angles between $\mathbf{r^n_{ij}}$ and $\mathbf{r^n_{ik}}$. Figure 1 shows the relationship between the vectors r_{ij}, r_{ik} and the angle α_{ijk} defined above. Γ_n uniquely defines a subset of configurations of Φ that are invariant under group translations and rotations.

It should be noted that if enough d_{ij}'s are specified to have just rigid configurations in Φ, then only the signs of the α^n_{ijk} are needed to define Γ_n. The following relationships are also satisfied:

$$\alpha^n_{ijk} = -\alpha^n_{ikj} \qquad \forall n,i,j,k \tag{3}$$

$$\pi = \alpha^n_{ijk} + \alpha^n_{jki} + \alpha^n_{kij} \qquad \forall n,i,j,k \tag{4}$$

Taking this into account a more restricted distance-based problem may be formulated as the following:

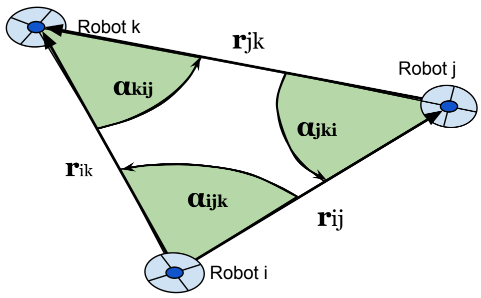

Fig. 1. Relationship between the relative position vectors $\mathbf{r_{ij}}$, $\mathbf{r_{ik}}$ and the angle α_{ijk}.

Definition 2. Given N omnidirectional robots defined by (1), find a control law for $\mathbf{u_i}, \forall i$ such that the configuration of the robots tend to a subset of Φ with a given and suitable Γ_n.

In the following a formulation to attempt to solve this problem is presented.

2.2 Combined attractive-repulsive artificial potential field

Artificial potential fields have been used for some time to help in the navigation problem. Most works usually use an attractive potential field to direct the robot to its goal and repulsive fields to avoid obstacles. When both fields depend only on the distances between robots a combined version may also be used [Ogren et al., 2004] [Dimarogonas and Johansson, 2009].

In this work the following logarithmic combined potential field is used:

Definition 3. Given two robots $\mathbf{q_i}$ and $\mathbf{q_j}$ with $j \in N_i$, an Artificial Potential Field φ_{ij} is defined as:

$$\varphi_{ij} = \frac{r_{ij} - d_{ij}}{d_{ij} - a_{ij}} - \log\left(\frac{r_{ij} - a_{ij}}{d_{ij} - a_{ij}}\right) \tag{5}$$

with $r_{ij} = |\mathbf{r_{ij}}|$ and $a_{ij} = \frac{1}{2}(\rho_i + \rho_j)$.

Figure 2 shows the shape of the combined potential field. Observed that φ_{ij} is well defined for $r_{ij} > a_{ij}$ and satisfies:

$$\begin{aligned} \varphi_{ij} &\geq 0, \quad \forall r_{ij} > a_{ij} \\ \varphi_{ij} &= \varphi_{ji}, \ \forall i,j \\ \varphi_{ij} &= 0 \quad \Leftrightarrow r_{ij} = d_{ij} \\ \lim_{r_{ij} \to a^+_{ij}} \varphi_{ij} &= \infty \end{aligned} \tag{6}$$

Using a gradient descent approach this potential function would help to drive the distance between robots to d_{ij} while avoiding contact between them. Besides, the following identity holds.

$$\frac{\partial \varphi_{ij}}{\partial r_{ij}} = \frac{1}{d_{ij} - a_{ij}} - \frac{1}{r_{ij} - a_{ij}} \tag{7}$$

3. FORMATION CONTROL

3.1 Distance based Formation control

The following control law has been proposed to solve the problem stated in Definition 1,

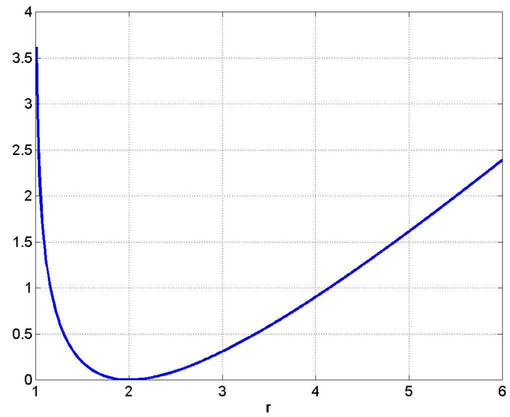

Fig. 2. Combined potential field as a function of r for the case where $d_{ij} = 2$ and $a_{ij} = 1$.

$$\mathbf{u_i} = -k_c \frac{\partial}{\partial \mathbf{q_i}} \varphi_i = -k_c \nabla_i \varphi_i \tag{8}$$

$$\varphi_i = \sum_{j \in N_i} \varphi_{ij} + \sum_{j \notin N_i} \gamma_{ij}$$

with φ_{ij} given by equation (5) and γ_{ij} given by a local repulsive potential field such as Khatib's [Khatib, 1986]. It has been shown that, by defining a Lyapunov function $V(\mathbf{q})$ given by,

$$V(\mathbf{q}) = \sum_i \varphi_i \tag{9}$$

its time derivative becomes,

$$\frac{dV(\mathbf{q})}{dt} = \sum_{i,j=1}^{N} \nabla_j \varphi_i \cdot \dot{\mathbf{q}}_\mathbf{j} = -\frac{2}{k_c} \sum_{i=1}^{N} \|\dot{\mathbf{q}}_\mathbf{i}\|^2 \tag{10}$$

Using LaSalle's Invariant Theorem the configuration of robots is proven to converge to a state where all robots stop. In general, this set equals Φ only when the graph associated to the formation is a tree as shown by [Dimarogonas and Johansson, 2008]. This result implies that the final configuration may not be suitable for some applications.

The following section introduces a control approach that uses angular information from the robots to try to avoid this problem.

3.2 Modified Formation Control with angular information added

For the case of the problem stated in definition (2) the following control law is proposed:

$$\mathbf{u_i} = -k_c \nabla_i \varphi_i - k_a \nabla_i \psi_i^n \tag{11}$$

$$\varphi_i = \sum_{j \in N_i} \varphi_{ij} + \sum_{j \notin N_i} \gamma_{ij} \tag{12}$$

$$\psi_i^n = \sum_{\substack{j,k \in N_i \\ j \neq k}} \psi_{ijk}^n \tag{13}$$

$$\psi_{ijk}^n = \alpha_{ijk}^n (\mathbf{r_{ij}} \times \mathbf{r_{ik}} \cdot \mathbf{k}) \Upsilon(\alpha_{ijk}^n (\mathbf{r_{ij}} \times \mathbf{r_{ik}} \cdot \mathbf{k}) + \theta) \tag{14}$$

with φ_{ij} and γ_{ij} the same as in equation (8), $\times$ refers to the cross product while $\mathbf{k}$ is the normal to the two dimensional space where the robots are in. The expression $(\mathbf{r_{ij}} \times \mathbf{r_{ik}} \cdot \mathbf{k})$ refers to the standard scalar triple product. $\Upsilon(\cdot)$ is the unit step function and $\theta > 0$ is a sufficiently small threshold to be designed.

The term $\alpha_{ijk}^n (\mathbf{r_{ij}} \times \mathbf{r_{ik}} \cdot \mathbf{k}) + \theta$ can also be thought of as a restriction on the robot configurations. When they are active a corrective action is applied to take the robot formation to a location where the usual distance based potential field will take the system to Φ.

A new Lyapunov function candidate $V_n(\mathbf{q})$ is defined as,

$$\begin{aligned} V_n(\mathbf{q}) &= \sum_i \varphi_i + \lambda \sum_i \psi_i^n \\ &= \sum_i \sum_{\substack{j,k \in N_i \\ j \neq k}} \left(\tfrac{1}{2(g_i-1)}(\varphi_{ij} + \varphi_{ik}) + \lambda \psi_{ijk}^n\right) + \sum_i \sum_{j \notin N_i} \gamma_{ij} \end{aligned} \tag{15}$$

with $\lambda > 0$ and g_i the number of robots in N_i. For $\theta = 0$ it is clear that $V_n(\mathbf{q}) > 0 \ \forall (q) \notin \Phi_n$. For $\theta > 0$ each term in the right hand side of equation (15) can be made positive with a sufficiently small selection of θ as long as Φ_n does not include a configuration with three robots in a line.

This proposition can be shown by noting that over the line connecting robots j and k the functions φ_{ij} and ψ_{ijk}^n are continuous and also,

$$\psi_{ijk}^n = 0 \tag{16}$$

$$\min_{\mathbf{q_i} | \mathbf{r_{ij}} \times \mathbf{r_{jk}} = 0} (\varphi_{ij} + \varphi_{ik}) > 0 \tag{17}$$

as long as $d_{ik} \neq d_{ij} + d_{jk}$. More specifically, after some mathematical computations omitted here, and assuming without loss of generality that $r_{ij} < r_{ik}$, it is possible to see that

$$\arg\min_{\mathbf{q_i} | \mathbf{r_{ij}} \times \mathbf{r_{jk}} = 0} (\varphi_{ij} + \varphi_{ik}) = \lambda_j \mathbf{q_j} + \lambda_k \mathbf{q_k} \tag{18}$$

with

$$\begin{aligned} \lambda_j &= \frac{1}{2}\left(1 + \frac{x}{r_{jk}}\right) \\ \lambda_k &= \frac{1}{2}\left(1 - \frac{x}{r_{jk}}\right) \\ x &= r_d + \sqrt{r_d^2 + r_p^2} \\ r_d &= \frac{(r_{ij} - d)(r_{ik} - d)}{r_{ij} + r_{ik} - 2d} \\ d &= \frac{1}{2}(\rho_j + \rho_k) \\ r_p &= r_{jk} + \frac{1}{2}(\rho_k - \rho_j) \end{aligned} \tag{19}$$

In the case when all d_{ij} are specified it is possible to show that $\dot{V}$ can be made negative semidefinitive.

$$\begin{aligned} \dot{V}_n(\mathbf{q}) &= \sum_i \dot{\varphi}_i + \lambda \sum_i \dot{\psi}_i^n \\ &= \sum_i \sum_l \nabla_l^t (\varphi_i + \lambda \psi_i^n) \cdot \dot{\mathbf{q}}_\mathbf{l} \\ &= \sum_l \sum_i \nabla_l^t (\varphi_i + \lambda \psi_i^n) \cdot (-k_c \nabla_l \varphi_l - k_a \nabla_l \psi_l^n) \end{aligned} \tag{20}$$

By substituting equations (12) and (13) into (20), the time derivative of the Lyapunov function transforms into,

$$\dot{V}_n(\mathbf{q}) = \sum_{l,i} \nabla_l^t \left(\sum_{j \neq i} \varphi_{ij} + \lambda \sum_{\substack{j,k \neq i \\ j \neq k}} \psi_{ijk}^n \right) \cdot \dot{\mathbf{q}}_{\mathbf{l}} \quad (21)$$

$$= \sum_{l,i,j} \bar{\delta}_{ij} \nabla_l^t \left(\varphi_{ij} + \lambda \sum_k \bar{\delta}_{jk} \bar{\delta}_{ki} \psi_{ijk}^n \right) \cdot \dot{\mathbf{q}}_{\mathbf{l}} \quad (22)$$

where

$$\bar{\delta}_{ij} = \begin{cases} 0 & i = j \\ 1 & \text{otherwise} \end{cases}$$

From equation (14) it can be shown that,

$$\nabla_l \psi_{ijk}^n = \begin{cases} \alpha_{ijk}^n \mathbf{k} \times \mathbf{r_{jk}} & l = i \\ \alpha_{ijk}^n \mathbf{k} \times \mathbf{r_{ki}} & l = j \\ \alpha_{ijk}^n \mathbf{k} \times \mathbf{r_{ij}} & l = k \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

Adding the symmetry properties of functions φ_{ij} and ψ_{ijk}^n, it follows that

$$\dot{V}_n(\mathbf{q}) = \sum_{lj} 2\bar{\delta}_{lj} \nabla_l^t \varphi_{lj} \cdot \dot{\mathbf{q}}_{\mathbf{l}} +$$
$$+ \lambda \sum_{ljk} \bar{\delta}_{lj} \bar{\delta}_{kl} \bar{\delta}_{jk} \left(\alpha_{ljk}^n + \alpha_{klj}^n + \alpha_{jkl}^n \right) \mathbf{k} \times \mathbf{r_{jk}} \cdot \dot{\mathbf{q}}_{\mathbf{l}}$$
$$= \sum_l \left(2\nabla_l^t \varphi_l + \lambda \pi \nabla_l^t \psi_l^n \right) \cdot \dot{\mathbf{q}}_{\mathbf{l}} \quad (24)$$

Choosing the control parameters such that $\lambda \pi k_c = 2k_a$, it becomes

$$\dot{V}_n(\mathbf{q}) = -2k_c \sum_l \|\nabla_l \varphi_l + \tfrac{1}{2} \lambda \pi \nabla_l \psi_l^n\|^2 \quad (25)$$

It should be noted that equation (25) implies that there are still unwanted local minima. However, by using larger values of λ, i.e. larger values of k_a the region of attraction of those values are being reduced. Besides, if the values of α_{ijk}^n are substituted in (14) by their signs only, the constant π is changed by a three in equations (24) and (25), maintaining the end result. The effect of θ is to remove the undesired configurations when robots are aligned. The transition occurs when $\alpha_{ijk}^n \mathbf{r_{ij}} \times \mathbf{r_{ik}} \cdot \mathbf{k} + \theta = 0$. This equation describes a line s parallel to $\mathbf{r_{jk}}$ and to a distance given by

$$\text{dist}\left(s, \mathbf{r_{jk}}\right) = \frac{\theta}{r_{jk}} \quad (26)$$

for small values of θ.

To illustrate the behavior, when $N = 3$, and for sufficiently large values of r_{ij} and r_{ik}, the proposed control law causes robot i to slide along s, since the velocity vector field points towards the opposite side as shown in figure 3.

4. SIMULATION RESULTS

To illustrate the proposed strategies, two simulations are presented for the formation control of three robots. In this case, a rigid formation implies all d_{ij} are given. The initial condition for the robots is as follows, $(x_1, y_1) = (-3, -3)$, $(x_2, y_2) = (3, -3)$, $(x_3, y_3) = (0, 3)$. The desired distances are set to $d_{ij} = 1$. The control parameters are set to: $k_c = 2, k_a = 3, \theta = 0.5$.

In the first simulation $\alpha_{123} = \alpha_{312} = -\alpha_{213} = -1$ to disable the restrictions and check the standard strategy.

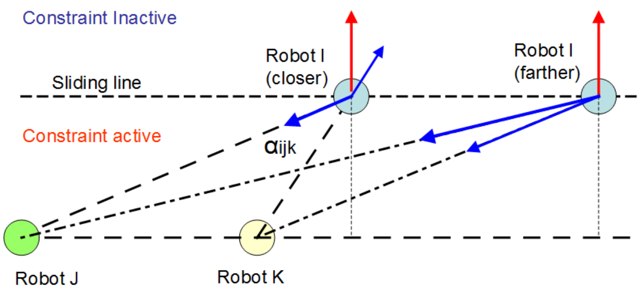

Fig. 3. Forces exerted over robot I when close to the sliding surface, near and far of robots J and K.

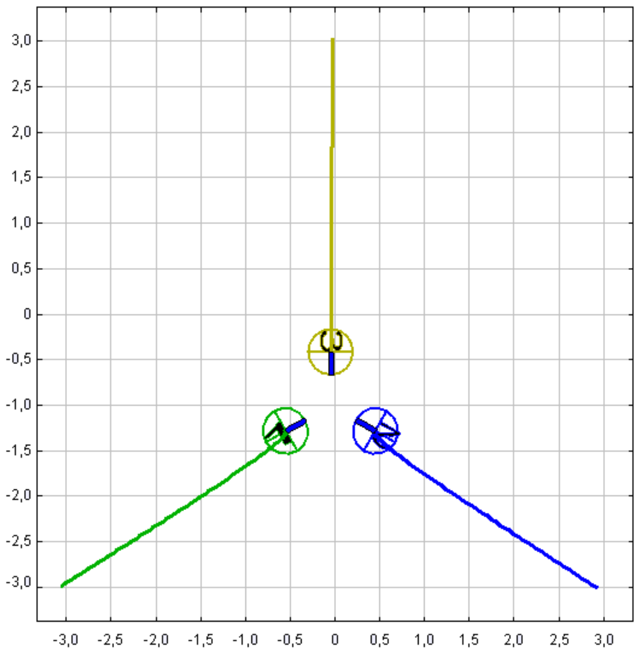

Fig. 4. Trajectories showing convergence to a formation with restrictions not active.

In this way Figure 4 shows a very smooth behavior, converging to the desired formation. Figures 5 and 6 show the control actions applied to all robots while figure 7 displays the profile of the Lyapunov function over the selected trajectory.

In the second simulation the restrictions become active by using $\alpha_{123} = \alpha_{312} = -\alpha_{213} = 1$. Figure 8 shows an initial period where robots 1 and 3 are exchanging positions to deactivate the constraints. After that, a smaller sliding region over s and finally the combined artificial potential takes over to reach the desired configuration pattern. Figure 9 shows the evolution of the distances between robots. Figures 10 and 11 show the control actions applied to all robots while figure 12 displays the profile of the Lyapunov function over the trajectory performed by the formation. Control actions are increased but the shape of the Lyapunov function is still smooth.

5. CONCLUSIONS

This work presents a formulation for distance based formation control of groups of omnidirectional robots aiming at reducing undesired local minima. The approach

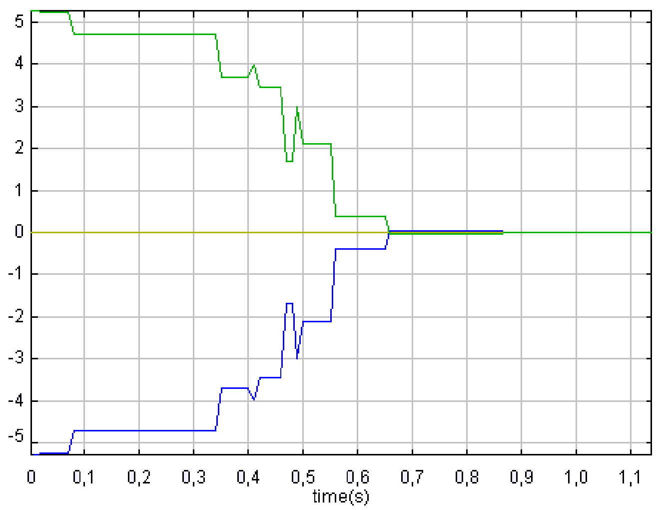

Fig. 5. Control actions, i.e. robot velocities in x axis, for all robots.

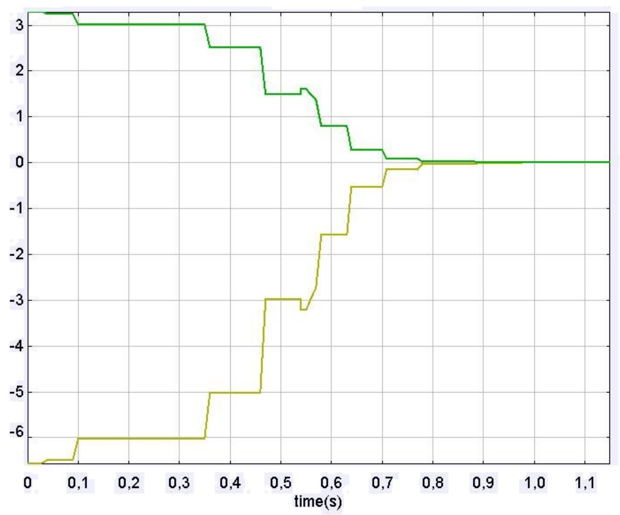

Fig. 6. Control actions, i.e. robot velocities in y axis, for all robots.

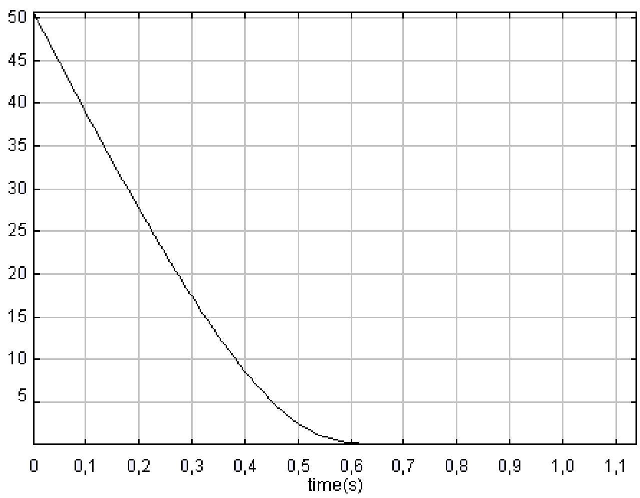

Fig. 7. Lyapunov function over time.

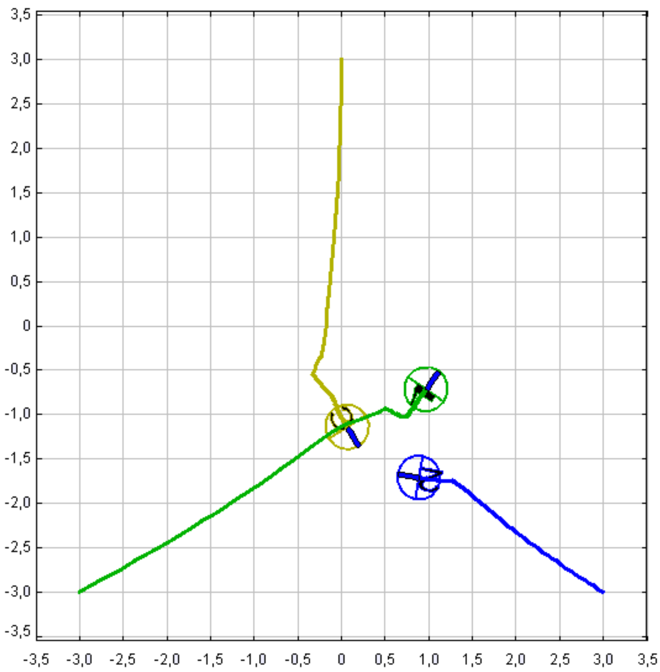

Fig. 8. Trajectories showing convergence to a formation with restrictions active.

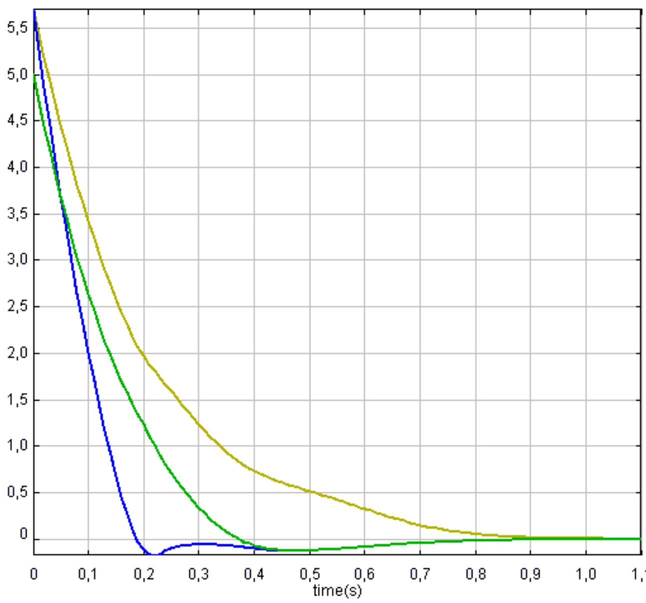

Fig. 9. Time evolution of inter-robot error distances with active restrictions.

uses a combined attractive-repulsive artificial potential field and a control term related to the angular information between robots. Lyapunov theory is used to show that the robots will converge to the desired configuration enlarging the region of attraction using suitable values for the angular control term. It should be noted that the potential field and angular control term used in this paper depend on local information and are suitable for application in decentralized algorithms. Simulations are presented for the case of three robots showing the behavior of the trajectories and control actions for the robots, as well as the shape of the Lyapunov function when the constraints are inactive and active. Extensions to prove the desired

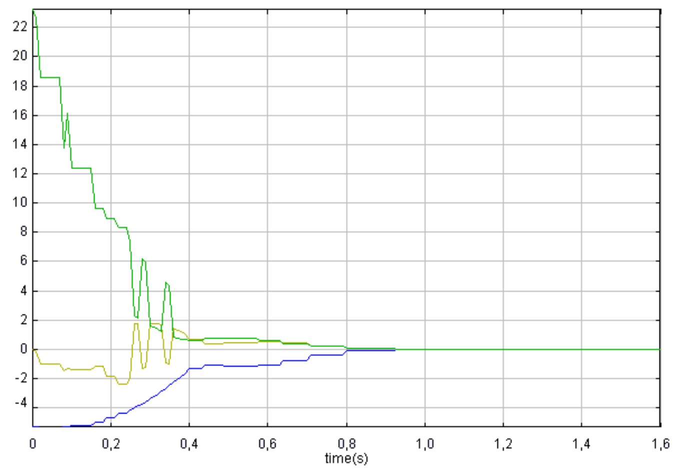

Fig. 10. Control actions, i.e. robot velocities in x axis, for all robots.

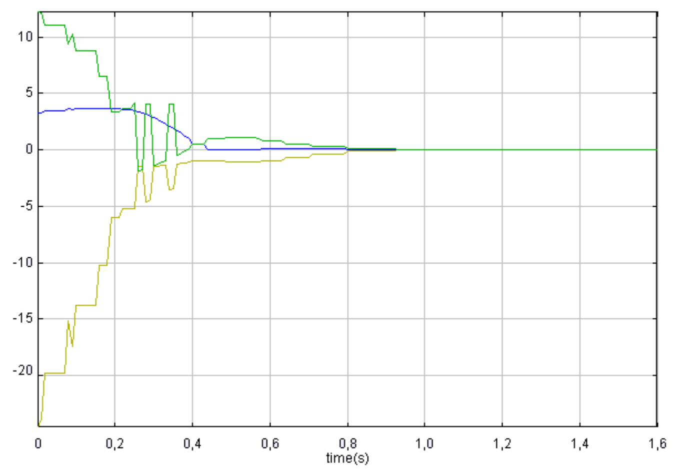

Fig. 11. Control actions, i.e. robot velocities in y axis, for all robots.

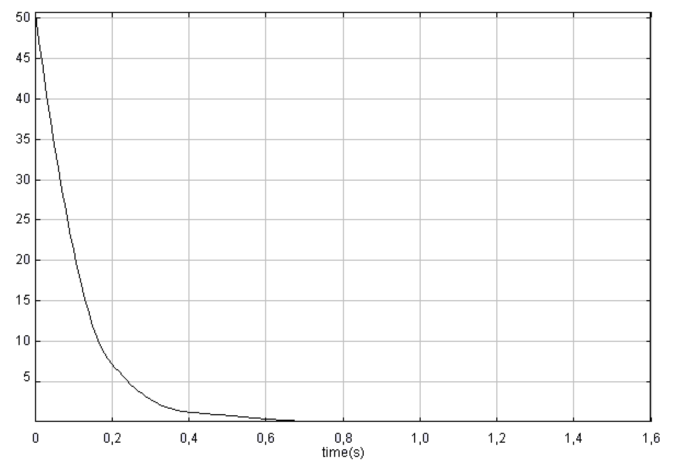

Fig. 12. Lyapunov function over time for simulation with active restrictions.

behavior for partial communication between robots, as well as introducing robot dynamics and marching control are subject of ongoing research.

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