The Treatment of Non-essential Inputs in a Cobb-Douglas Technology

An Application to Mexican Rural Household Level Data

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Abstract:

The standard approach for fitting a Cobb-Douglas production function to micro-data with zero values is to transform zero-values to facilitate the logarithmic transformation. In general the estimates obtained are extremely sensitive to the transformation chosen, generating doubts about the use of a specification that assumes all inputs are essential (as the Cobb-Douglas does) when that is not the case. An alternative method is presented in the paper which allows to actually estimate the degree of essentiality of the various production inputs, retaining at the same time the Cobb-Douglas specification. By utilizing the properties of translatable homothetic functions, I estimate by how much the origin of the input set should be translated to allow for the Cobb-Douglas functional form to capture the fact that the data have positive amount of output even when some of the inputs are not being used. The approach is applied to Mexican farm level production data collected by the author. Many households did not use family or hired labor on farm production, or had different capital composition. An important feature of the estimations is that they provide a clear measurement of the degree of essentiality of potentially non-essential inputs and also an indication of the size of the error introduced by the common "trick" of adding a "small" value to zero input values.

Key Words: Translation homotheticity, production function, non-essential inputs.

1. Introduction

Cobb-Douglas functions are among the best known production functions utilized in applied production analysis[1]. The most general form for a Cobb-Douglas is: $f(x) = A\prod_{i=1}^{n} x_i^{b_i}$, $b_i > 0,\ i = 1,2,...,n.$

This functional form has the properties of:

i) strict monotonicity: if $x' > x$ then $f(x') > f(x)$;

III) quasi-concavity: $V(y) = \{x : f(x) \geq y\}$ is a convex set ;

IV) strict essentiality: of $f(x_a,...x_{i-1},0,x_{i+a},...,x_n) = 0$ *for all* $x_i > 0$;

iv) the set $V(y)$ is closed and nonempty for all y>0;

v) $f(x)$ is finite, nonnegative, real valued, and single valued for all nonnegative and finite x; it is also continuos and everywhere twice-continuously differentiable.

vi) $f(x)$ is homogenous of degree $k = \sum_{i=1}^{n} b_i$,

Property IV) indicates the Cobb-Douglas technology requires all inputs to be essential in production: all must be used in strictly positive amounts to obtain a positive output (i.e., the input requirement sets do not intersect the axis). This requirement of the production function is easily fulfilled when aggregated data—say country or industry level—are used. But, when a more micro level analysis is required, the researcher may well end up having some observations with positive levels of output, even when some of the inputs have zero values. This situation is typically found in analysis of labor supply in rural settings where, for instance, researchers need to differentiate household labor supply for farming by type of household member (e.g., male/female). As not all households use both types of labor for farming activities, some observations have positive level of output but zero use of one (or both) of these inputs. That is to say, one (or both) of these particular inputs is non-essential for production. The same situation may show up when the researcher wants to concentrate his/her analysis in other inputs, as some farmers may not use them in production (e.g., hired labor, children labor, fertilizers, machinery). For these cases, a Cobb-Douglas (or the more general translog) can be used only if we make some transformation to the zero-value arguments[2].

Researchers in general estimate a logarithmic transformation of (1) in the form:

(2) $\ln(y(x)) = \ln A + \sum_{i=1}^{n} b_i \ln(x_i), \quad b_i > 0,\ i = 1,2,...,n.$

and modified zero-value arguments by either replacing them by 1--that is $\ln(x_i) = 0$ when $x_i = 0$--or with "small" values (see, for instance, MaCurdy and Pencavel, 1986, and Jacoby, 1992). In other words, whenever they find inputs that are non-essential (i.e., for some observation i $y_i > 0$ but $x_{ki} = 0$) they replace x_{ki} by $x'_{ki} = x_{ki} + a_i$, with a_i equal to 1—or to a "sufficiently small" value—using the same value for all i (i.e., $a_i = \bar{a}$). Obviously, these procedures are arbitrary and are forcing the production function to include input quantities that are not actually observed. I show in the following empirical section of the paper that changes in the $\bar{a}$ values adopted may cause the estimated regression coefficients and their standard errors to vary significantly,

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[1] The present analysis is centered in the Cobb-Douglas functional form only for exposition purposes. The same analysis carries over other, more general, functional forms (e.g., the translog, that can be restricted to obtain the Cobb-Douglas) (see Chambers, 1988).
[2] The estimation of production functions in general, and Cobb-Douglas production functions in particular, presents many additional problems. See Varian, 1984, Chapter 4, *Econometrics and Economic Theory*, for a discussion.

generating doubts about the “tricks” used to retain a specification that implies that all inputs are essential (as the Cobb-Douglas does) when that is not the case. This paper proposes an alternative method which uses the properties of translation homotheticity, and translates the origin of coordinates of the production space in the direction of the non-essential inputs. The translation coefficients are estimated by maximum likelihood.

I highlight the empirical importance of the approach by applying it to farm level production data coming from a World Bank 1995 survey I conducted in rural Mexico. Table 1 presents the mean value of key variables of the data. As many households in the data did not use family labor on farm production, or did not use hired labor, and had different capital composition (some zero non-land farm assets), the sample provides good testing ground to see the effect of the alternatives ways of “solving” the problem posit by inputs with zero values. An important feature of the estimations is that they provide a clear measurement of the degree of non-essentiality of all non-land inputs.

In what follows, I assess the impact on the estimates of different assumptions about a_i when a Cobb-Douglas production function is estimated with farm level data. I then apply the new procedure developed in this paper to the same data set and compare them with those of the previous sections. The last part of the paper summarizes the findings.

Table 1. Descriptive statistics

		STATE			
Variable	Units	Guanajuato	Sonora	Puebla	Tlaxcala
Value of output per hectare	US$ of 1994	691	974	482	363
Production factors					
Land planted	Hectares	15.1	35.7	3.6	4.6
Value of non-land assets	US$ of 1994	51911	151246	13729	9010
Value of animal assets	US$ of 1994	8378	89466	5013	4589
Expenditures on hired labor	US$ of 1994	16993	29012	1493	1796
Expenditures on other inputs	US$ of 1994	18680	33508	801	1447
Family labor, adult male>12	%	1.6	1.0	1.5	1.4
Family labor, adult female	%	0.9	0.1	0.9	0.7
Household's demographics					
# Children<13/# adults	%	0.47	0.4	0.45	0.46
Education male family labor	Years	4.07	6.13	5.87	5.18
Education female family labor	Years	3.32	3.88	4.87	4.22
Age HH head	Years	59.54	58.06	58.1	57.04
Formal education HH head	Years	1.88	4.38	3.6	2.97
Proportion of male hh head	%	0.90	0.87	0.86	0.86
Proportion of HH head w/ off-farm jobs	%	0.12	0.25	0.37	0.38
Proportion of HH w/secure title on land	%	0.89	0.9	0.83	0.94
Location factors					
Distance to market	Km	8.89	23.16	14.2	8.71
Celaya 1	% hh in State	0.34	0	0	0
Celaya 2	% hh in State	0.34	0	0	0
Irapuato 1	% hh in State	0.17	0	0	0
Irapuato 2	% hh in State	0.16	0	0	0
Puebla 1	% hh in State	0	0	0.15	0
Puebla 2	% hh in State	0	0	0.27	0
Puebla 3	% hh in State	0	0	0.35	0
Puebla 4	% hh in State	0	0	0.23	0
Tlaxcala 1	% hh in State	0	0	0	0.48
Tlaxcala 2	% hh in State	0	0	0	0.52
Navojoa	% hh in State	0	0.48	0	0
Obregon	% hh in State	0	0.52	0	0

2. An example for farm level production data.

A more general form of expressing equation (2) would be

$$(3)\ \ln(y) = b_0 + \sum_{i=1}^{k} b_i \ln(x_i + a_i) + \sum_{j=1}^{n} b_j \ln(x_j + a_j) + m$$

where, for a given sample of data, inputs of type x_i are assumed to be positive for all observations and inputs of type x_j are assumed to take the value zero for some observations. That is to say, for a particular sample of data, "x_i type" of inputs are essential for production whereas "x_j type" are not. The econometric issue is to estimate the parameters b_0, b_i's, b_j's and s^2. In order to do that, the common procedure is to assume values for all the a_j's since otherwise the logarithm will not be defined for those observations with zero value for x_j. The key contribution of this paper is, instead of choosing beforehand the value for the a_j's, to "let the data tell us" what those values are by estimating them with a maximum likelihood technique. Although all x_i are assumed to be positive (which implies that $\ln(x_i)$ is always defined, even if it is assumed that all a_i's are zeroes), all the inputs, not just the x_j 's, have the possibility of being non-essential. To incorporate this, the methodology developed here allows also to estimate the values for the a_i's.

The following estimation uses 399 observations of the 1995 survey for which all the information required for estimating an agricultural Cobb-Douglas production function were available.

Descriptive statistics of this sub-sample of the data are presented in Table 1, and the proportion of the observations with some factors having zero values is detailed in Table2.

Table 2. Importance of inputs with zero values

Variable	# of times have value=0	% of total sample
Expenditures in hired labor	37	9
Family labor used in farming activities, males	45	11
Family labor used in farming activities, female	238	60
Non-Land Productive Assets (Machinery, etc.)	53	13
Assets in Animals	0	0
Expenditures in other inputs	0	0

Table 3 presents the results coming from estimating equation (3) under different assumptions about the translation parameters a_i and a_j, and highlights the problems of using the ad-hoc solutions indicated in the introduction to this paper. The table has four sets of estimates: the first after adding "1" to the variables with some zero values, the second after adding "0.1", the third after adding "0.01" and the fourth after adding "0.001"[3]. The R's-squared indicate a good fit of the model (around 83%), and the sign of the "production factors" variables are positive as expected. The quantity of hectares planted, the non-land assets, the expenditures on hired labor and expenditures on other inputs were statistically significant in the four estimations as well as male family labor applied to agriculture (except in the first regression). Returns to scale are about constant in the four regressions, which is in line with other studies done on agricultural production (López and Valdés, 1998).

The last three columns of the table summarize the results that are of importance for the purpose of this paper. Under the heading of “Range” I calculated the difference between the highest and the lowest of the parameters estimated in the four regressions. In the penultimate and the last column of the table I calculated the ratio of the “Range” to the Max and the min values of each regressor respectively. It is clear from these columns that “important” coefficients of the regression vary significantly according to the value of the $\bar{a}$ chosen: the marginal productivity of *non-land assets* ranges from a minimum of 0.024 (when $\bar{a}$ =0.001) to a maximum of 0.045 (when $\bar{a}$ =1), which implies 47% of the maximum value of the parameter (or 88% of the minimum). Similar percentages can be found in the case of the estimates for *expenditures on hired labor*. For the case of *male family labor applied to agriculture* the coefficient was not significant in the first regression and turned out statistically significant in the other three with a wide range of variation in the estimated value of the parameter (from a Max of 0.113 to a min of 0.049.

[3] The a_j 's are equal to $\bar{a}$, that is, the same for all *j*'s, whereas the a_i 's are implicitly assumed to be zeroes. Researchers’ choice of $\bar{a}$ is acknowledged to be arbitrary. For the purpose of this paper, I present here a set of four “small” values, which are those usually found in empirical papers.

Table 3. Alternative procedures generally used for non-essential inputs

Variable	If some $x_{ki}=0$, $\log(x_k)=\log(x_k+1)$			If some $x_{ki}=0$, $\log(x_k)=\log(x_k+0.1)$			If some $x_{ki}=0$, $\log(x_k)=\log(x_k+0.01)$			If some $x_{ki}=0$, $\log(x_k)=\log(x_k+0.001)$			Max	Min	Range	Range/ Max	Range/ Min
	Estim.	Std.err	Sig. (1)	Estim.	Std.err	Sig. (1)	Estim.	Std.err	Sig. (1)	Estim.	Std.er	Sig. (1)	(a)	(b)	(c)= (a)-(b)	(c)/(a)	(c)/(b)
Intercept	3.218	0.905	***	3.085	0.904	***	3.150	0.907	***	3.207	0.910	***	3.218	3.085	0.133	4%	4%
Production factors																	
Fam-lab-adult male	0.066	0.092		0.113	0.052	**	0.070	0.031	**	0.049	0.021	**	0.113	0.049	0.065	57%	133%
Fam-lab-adult-female	0.001	0.126		0.007	0.062		0.007	0.035		0.006	0.024		0.007	0.001	0.006	83%	479%
Ha planted	0.386	0.060	***	0.390	0.060	***	0.400	0.060	***	0.407	0.060	***	0.407	0.386	0.021	5%	5%
Assets	0.045	0.013	***	0.036	0.011	***	0.029	0.009	***	0.024	0.008	***	0.045	0.024	0.021	47%	88%
Animal stock	0.007	0.011		0.006	0.011		0.006	0.011		0.006	0.011		0.007	0.006	0.002	25%	33%
Expend-labor	0.093	0.019	***	0.073	0.015	***	0.057	0.012	***	0.047	0.010	***	0.093	0.047	0.046	49%	97%
Expend-inputs	0.411	0.042	***	0.429	0.042	***	0.437	0.042	***	0.443	0.042	***	0.443	0.411	0.031	7%	8%
Household's demographics																	
#Children/# adults	0.212	0.084	***	0.197	0.084	**	0.189	0.085	**	0.186	0.085	**	0.212	0.186	0.027	13%	14%
Education wkland male	0.029	0.057		-0.095	0.081		-0.113	0.083		-0.116	0.083		0.029	-0.116	0.145	497%	-125%
Education wkland fem.	-0.151	0.064	**	-0.160	0.101		-0.165	0.104		-0.167	0.104		-0.151	-0.167	0.016	-10%	-9%
Hh head has other job	-0.022	0.093		-0.020	0.092		-0.019	0.093		-0.017	0.093		-0.017	-0.022	0.005	-29%	-22%
Education hh head	0.127	0.067	*	0.175	0.069	***	0.186	0.069	***	0.190	0.069	***	0.190	0.127	0.063	33%	49%
Age hh head	-0.064	0.199		0.053	0.205		0.075	0.207		0.080	0.207		0.080	-0.064	0.144	181%	-224%
Hh is male headed(dummy)	0.197	0.130		0.169	0.130		0.173	0.130		0.176	0.130		0.197	0.169	0.029	15%	17%
Land is titled (dummy)	0.212	0.123	*	0.210	0.122	*	0.206	0.122	*	0.204	0.123	*	0.212	0.204	0.008	4%	4%
Location factors																	
Distance to market	-0.001	0.028		-0.006	0.028		-0.007	0.028		-0.008	0.028		-0.001	-0.008	0.006	-484%	-83%
Pue1 (dummy)	-0.085	0.255		-0.092	0.253		-0.076	0.254		-0.064	0.254		-0.064	-0.092	0.028	-43%	-30%
Pue2 (dummy)	0.722	0.182	***	0.755	0.182	***	0.773	0.182	***	0.783	0.182	***	0.783	0.722	0.061	8%	8%
Pue3 (dummy)	0.222	0.170		0.213	0.170		0.221	0.170		0.227	0.170		0.227	0.213	0.014	6%	7%
Pue4 (dummy)	0.659	0.203	***	0.692	0.202	***	0.695	0.203	***	0.698	0.203	***	0.698	0.659	0.039	6%	6%
Irapua1 (dummy)	0.063	0.180		0.047	0.180		0.045	0.180		0.044	0.181		0.063	0.044	0.019	30%	43%
Irapua2 (dummy)	0.681	0.199	***	0.674	0.200	***	0.683	0.200	***	0.690	0.200	***	0.690	0.674	0.016	2%	2%
Celaya1 (dummy)	0.728	0.152	***	0.710	0.152	***	0.709	0.153	***	0.710	0.153	***	0.728	0.709	0.019	3%	3%
Celaya2 (dummy)	0.497	0.149	***	0.486	0.148	***	0.494	0.149	***	0.500	0.149	***	0.500	0.486	0.014	3%	3%
Obregon (dummy)	0.662	0.186	***	0.667	0.186	***	0.670	0.186	***	0.673	0.187	***	0.673	0.662	0.011	2%	2%
Navojoa (dummy)	0.903	0.173	***	0.915	0.173	***	0.919	0.173	***	0.921	0.174	***	0.921	0.903	0.018	2%	2%
Returns to scale	1.010			1.054			1.006			0.981			1.054	0.981	0.073	7%	7%
Adj-r-square	0.830			0.831			0.830			0.829							

All variables are in logs, except dummies. (1) Significance levels: *** at 99%, ** at 95%, * at 90%

The methodology used in this paper overcomes this problem by “letting the data tell us” what the values of the $\hat{a}_j$'s are. The basic principle behind the approach is Marc Nerlove’s dictum: "If it matters, it can be estimated". This was done in the empirical part of the model by obtaining maximum likelihood estimates of the a_i's and a_j's, in addition to b_0, b_i's, b_j's and s^2. That is, I estimated those values of the unknown parameters that would, under a multivariate normal specification, maximize the probability of obtaining the sample actually observed (Judge *et al.*, 1988, p. 222).The estimated parameters $\widehat{b}_0, \widehat{b}_i$'s, and $\widehat{b}_j$'s are the usual ones for a Cobb-Douglas technology, and $\hat{a}_i$'s and $\hat{a}_j$'s are the translation parameters for this particular case.

Results of maximum likelihood joint estimation, are presented in Table 4. The last two columns of the table highlights the differences with the estimates presented in Table 4. With arbitrary $\overline{a}$ ’s, some “production factors” estimates are always above what they should be: the coefficient for male family labor in agriculture is between 1.59 and 3.67 times bigger, and the one for hectares planted is more than 1.87 times bigger.
As the returns to scale are about constant also for this specification, the coefficients for the other “production factors”, female family labor, non-land assets, expenditures on hired labor, and expenditures on other inputs, are smaller— between 12% and 95% of the value of the estimates coming from our maximum likelihood method.

Table 4. Maximum likelihood estimation of all alphas.

Variables	Estimates (a)	Std.error	Significance (1)	(a) from table 3)/(a), in %	(b) from table 3)/(a), in %
Intercept	1.754	0.909	**	184	176
Production Factors					
Fam-Lab-Adult Male	0.031	0.009	***	367	159
Fam-Lab-Adult-Female	0.009	0.015		82	12
Ha Planted	0.207	0.051	***	197	187
Assets	0.073	0.019	***	62	33
Animal Stock	0.009	0.012		78	67
Expend-Labor	0.188	0.035	***	49	25
Expend-Inputs	0.466	0.043	***	95	88
Household's Demographics					
#Children/# Adults	0.262	0.083	**	81	71
Education Wkland Male	-0.096	0.080		-30	121
Education Wkland Female	-0.164	0.099	*	92	102
Hh Head Has Other Job	-0.057	0.089		30	39
Education Hh Head	0.174	0.068	***	109	73
Age Hh Head	0.180	0.203		44	-36
Hh Is Male Headed (Dummy)	0.244	0.124	**	81	69
Land Is Titled (Dummy)	0.187	0.118		113	109
Location Factors					
Distance To Market	-0.002	0.032		44	351
Pue1 (Dummy)	-0.155	0.241		41	59
Pue2 (Dummy)	0.571	0.179		137	126
Pue3 (Dummy)	0.176	0.164	***	129	121
Pue4 (Dummy)	0.465	0.193		150	142
Irapua1 (Dummy)	0.014	0.172	**	440	307
Irapua2 (Dummy)	0.615	0.194		112	110
Celaya1 (Dummy)	0.666	0.145	***	109	106
Celaya2 (Dummy)	0.458	0.144	***	109	106
Obregon (Dummy)	0.689	0.178	***	98	96
Navojoa (Dummy)	0.902	0.166	***	102	100
Alphas					
Alpha-Family Male Labor	0.000000107	0.000000039			
Alpha-Family Female Labor	0.00001317	0.0000519			
Alpha-Non-Land Assets	72.220	97.067			
Alpha-Expenditures hired labor	102.560	56.348	*		
Alpha-Animal assets	18.329	19.468			
Alpha-Other expenditures	4.990	36.863			
Returns To Scale	0.983			107	100
Adj-R-Square (From OLS)	82.98				

(1) Significance levels: *** at 99%, ** at 95%, * at 90%

Thus, if, for example, we use the marginal productivity of family labor force in agriculture activities to assess family labor allocation to off-farm activities (as in Jacoby-1992- for instance), we are going to overestimate its true on-farm productivity (by 59% or 267%, depending on the $\bar{a}$ chosen).

Likelihood ratio tests rejected at the 99%,99%,95% and 90% significance level the null hyphoteses that the estimated alphas of table 5 are statistically the same to those used in any of the four exercises of table 3, respectively[4].

The alphas estimated are an indication of the degree of essentiality of the production inputs. This can be seen more clearly in Table 5, which contains the estimated value of the alphas, the sample mean of the variable they are attached to, and the ratio of these two values. Results shows that the ordering of the inputs taking into account their degree of essentiality is: Male family labor, female family labor, other expenditures, farm assets, expenditures on hired labor, and, animal assets[5]. That means that, for instance, it is "more difficult" to have some positive level of production without male family labor in agriculture than without female family labor in agriculture. In turn, it is relatively easier to get some production when animal assets are zero than when the other forms of non-land farm assets are zero, since the origin of the input set was translated in the direction of the latter inputs by 0.138% whereas for animal assets the translation was 0.491%. It is important to notice that since only the alpha for hired labor is significantly different from

[4] When the alphas are estimated , the maximized value of the log likelihood function is -435.693. When the alphas are assumed to be fixed, the values are -445.982,-443.771, -442.386 and -441.093 for values of alpha 1, 0.1, 0.01 and 0.001 respectively.
[5] The ranking starts with the smaller translation of the input set in the direction of the input. For instance, for male family labor the translation is only 0.00001% of the sample mean of the variable, closer to the origin than, for example, female family labor (translated by 0.002% of its sample mean).

zero, in an statistical sense only this input is truly "non-essential". As they are data-specific, alpha values are most likely going to vary when this procedure is applied to different a different data set.

Table 5. Relative importance of the alphas

Alpha:	Estimated coeff. (a)	Sample mean of the variable alpha is attached to. (b)	(c)=(a)/(b), in %	Ranking
Male family labor in agriculture	0.00000011	1.4	0.00001	1
Female family labor in agriculture	0.000013	0.7	0.002	2
Non-land, non-animal farm assets	72.2	52364	0.138	4
Animal assets	102.6	20893	0.491	6
Expenditures on hired labor	18.3	12732	0.144	5
Other expenditure	4.99	13990	0.036	3

3. Conclusions

The standard approach for fitting a Cobb-Douglas production function to micro-data with zero values is to transform zero-values to facilitate the logarithmic transformation. In general the estimates obtained are extremely sensitive to the transformation chosen, generating doubts about the use of a specification that assumes all inputs are essential (as the Cobb-Douglas does) when that is not the case. I propose here an alternative method which allows to actually estimate the degree of essentiality of the various production inputs, retaining at the same time the Cobb-Douglas specification. By utilizing the properties of translatable homothetic functions, I estimate by how much the origin of the input set should be translated to allow for the Cobb-Douglas functional form to capture the fact that the data have positive amount of output even when some of the inputs are not being used. To highlight the empirical importance of the approach, it is applied to farm level production data collected in rural Mexico. Many households did not use family labor on farm production, did not use hired labor, or had different capital composition (i.e., zero value for non-land farm assets). An important feature of the estimations is that they provide a clear measurement of the degree of essentiality of potentially non-essential inputs and also an indication of the size of the error introduced by the common "trick" of adding a "small" value to zero input values.

Appendix: Translation homotheticity

Chambers and Färe (p. 632) introduced the concept of translation homotheticity. The technology structure is translation homothetic if $L(y)$ can be written as

(3) $L(Y) = H(y; g_x)g_x + L(1), \quad y \in \Re_+^M$, where

$L(y) = \{x : x \text{ can produce } y\}, \quad y \in \Re_+^M$;

$L(1) = \{x : x \text{ can produce 1, which is the reference output vector}\}$, and

$H(y;.)$ is a nondecreasing function consistent with the following properties:

a.- $\vec{D}_i(y, x - ag_x; g_x) = \vec{D}_i(y, x; g_x) - a, \; a \in \Re;$

b.- $\vec{D}_i(y, x; lg_x) = \frac{1}{l}\vec{D}_i((y, x; g_x), \; l > 0;$

c- $(x', -y') \geq (x, -y) \Rightarrow \vec{D}_i(y', x'; g_x) \geq \vec{D}_i(y, x; g_x);$
i.e., non decreasing in inputs and nonincreasing in output;

d.- $\vec{D}_i(y, x; g_x)$ is concave in x;

e.- $x \in L(y) \Leftrightarrow \vec{D}(y, x; g_x) \geq 0.$

Where $\vec{D}_i$ is the directional input distance function developed by Chambers, Chung and Färe (1996), as is defined $\vec{D}_i : \Re_+^M \times \Re_+^N \times \Re^N \rightarrow \Re$ by

$$\vec{D}_i(y, x; g) = \sup_a \{a \in \Re : x - ag \in L(y)\};$$
$$= \sup_a \{a \in \Re : x \in ag + L(y)\}$$

Translation homotheticity can be visualized as having inputs sets for different output levels that are generated by taking a common reference set $L(1)$, and then translating that reference set in the direction of the vector g_x. A movement out from any point on $L(1)$ in the direction of g_x will cut isoquants at points having the same marginal rate of substitution as at the point on $L(1)$.

Figures 1a and 1b of Chambers, *et al.* (1996 p.410) are reproduced here to illustrates the concept. In Figure 1a $x \in L(y)$ and $\vec{D}_i(y, x; g)$ is given by the ratio

Figure 1a.

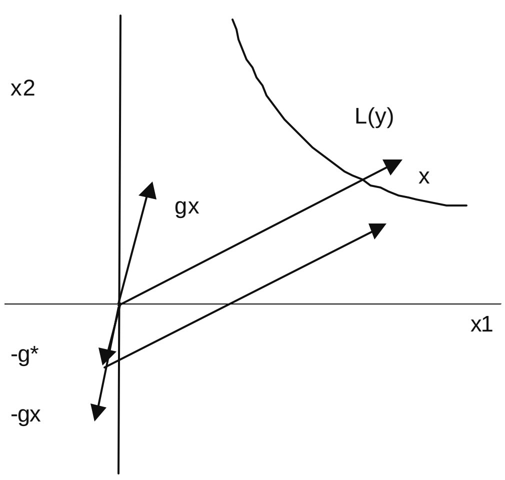
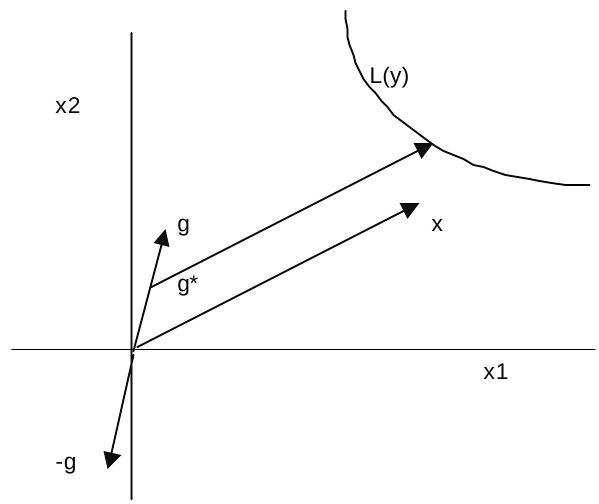


Figure 1b.



$\|g^*\|/\|g\| > 0$, where ||*k*|| denotes the norm of vector *k*. In Figure 1b $x \notin L(y)$ but moving x in the direction of g eventually encounters $L(y)$. Here $\vec{D}_i(y, x; g)$ is given by $-\|g^*\|/\|g\| < 0$.

In this paper, I use the properties of homothetic translatable production functions to handle non-essential inputs in a Cobb-Douglas. By (3),the production function can be expressed as the sum of the reference output $L(1)$ and the directional distance. Let us assume that from the n[th] dimensional vector of inputs $x = x(x_1,..., x_j, x_k,..., x_n)$,for some observations with $y > 0$ the inputs x_j and x_k are non-essential. I can choose the reference vector g_x in the direction of these non-essential inputs, and translate the origin of coordinates $x = x(0,..., 0,0,,... 0)$ to $x^* = x(0,..., a_j, a_k,...,0)$ in such a way that we can obtain positive amount of output with zero quantities of the inputs x_j and x_k , and while doing this, still conserve the Cobb-Douglas functional form. The new input set for the t[th] observation will be defined by $x_t^* = x(x_{1t},..., x_{jt} + a_j, x_{kt} + a_k,..., x_{nt})$. The a_j's and a_k's would provide a measurement of how non-essential are these non-essential inputs.

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